Extra Credit Rocks

Sign up for a Discover® Student Card today and enjoy:

- 0% Intro APR* on Purchases for 6 Months
- No Annual Fee
- Easiest Online Account Management Options
- Full 5% *Cashback Bonus*®* on Get More purchases in popular categories all year
- Up to 1% *Cashback Bonus*®* on all your other purchases
- Unlimited cash rewards that never expire as long as you use your Card

APPLY NOW

DISCOVER
CARD

*View Discover® Card Rates, Fees, Rewards and Other Important Information.

CHAPTER

3 Exponents and Polynomials

The amount A of a radioactive substance remaining after time t can be found using the formula $A = A_0(0.5)^{t/n}$, where A_0 represents the original amount of radioactive material and n is the half-life given in the same units of time as t. If the half-life of radioactive carbon 14 is 5,770 years, how much radioactive carbon 14 will remain after 11,540 years if we start with 100 grams?

3–1 ■ Properties of exponents

Exponential form

In chapter 1, we discussed exponents as related to real numbers. Since variables are symbols for real numbers, we shall now apply the properties of exponents to them. The expression a^3 (read "a to the third power") is called the **exponential form** of the product

$$a \cdot a \cdot a$$

We call a the **base** and 3 the **exponent.**

$$\text{Exponential form} \longrightarrow \overset{\text{Exponent}}{\underset{\text{Base}}{4^3}} = \underbrace{4 \cdot 4 \cdot 4}_{\text{3 factors of 4}} = 64 \qquad \text{Standard form}$$

Definition of exponents

$$a^n = \underbrace{a \cdot a \cdot a \cdots a}_{n \text{ factors of } a}$$

where n is a positive integer.

Concept

The exponent tells us how many times the base is used as a factor in an indicated product.

Note An exponent acts only on the symbol immediately to its left. That is, in xy^3, the exponent 3 applies only to the base y, whereas $(xy)^3$ would mean the exponent applies to both the x and the y.

Example 3–1 A

Write each expression in exponential form and identify the base and the exponent.

1. $x \cdot x \cdot x \cdot x = x^4$ — Base x, exponent 4
2. $4 \cdot 4 \cdot 4 = 4^3$ — Base 4, exponent 3
3. $(x^2 + y)(x^2 + y)(x^2 + y) = (x^2 + y)^3$ — Base $(x^2 + y)$, exponent 3
4. $(-3) \cdot (-3) \cdot (-3) \cdot (-3) = (-3)^4$ — Base -3, exponent 4
5. $-(3 \cdot 3 \cdot 3 \cdot 3) = -3^4$ — Base 3, exponent 4

Note In examples 4 and 5, we review the ideas of exponents related to signed numbers. Recall that $(-3)^4 = 81$, whereas $-3^4 = -81$.

▶ ***Quick check*** Write $(-5) \cdot (-5) \cdot (-5)$ in exponential form and identify the base and the exponent.

Product property of exponents

Consider the indicated product of $a^2 \cdot a^3$. If we rewrite a^2 and a^3 by using the definition of exponents, we have

$$a^2 \cdot a^3 = \overbrace{a \cdot a}^{a^2} \cdot \overbrace{a \cdot a \cdot a}^{a^3}$$

and again using the definition of exponents, this becomes

$$a^2 \cdot a^3 = \overbrace{a \cdot a \cdot a \cdot a \cdot a}^{\text{5 factors of } a} = a^5$$

This leads us to the observation that

Add exponents

$$a^2 \cdot a^3 = a^{2+3} = a^5$$

Multiply like bases

Base remains unchanged

Product property of exponents

For all real numbers a and positive integers m and n,

$$a^m \cdot a^n = a^{m+n}$$

Concept

When multiplying factors having **like bases,** add the exponents to get the exponent of the common base.

Example 3–1 B

Find each of the following products.

1. $y^4 \cdot y^5 = y^{4+5} = y^9$

2. $3^2 \cdot 3^4 = 3^{2+4} = 3^6 = 729$

Note When we multiply expressions of the same base, we add the exponents, we do not multiply the bases.

$$3^2 \cdot 3^4 \neq 9^6$$

3. $x \cdot x^7 \cdot x^3 = x^{1+7+3} = x^{11}$

Note If there is no visible exponent associated with a numeral or a variable, the exponent is understood to be 1.

4. $x^{3n} \cdot x^{2n} = x^{3n+2n}$ Multiply like bases by adding the exponents
$= x^{5n}$

▶ ***Quick check*** Multiply $a^2 \cdot a \cdot a^5$

Power of a power property of exponents

A second property of exponents can be derived by applying the definition of exponents and the product property of exponents. Consider the expression $(a^4)^3$.

$$(a^4)^3 = \underbrace{a^4\, a^4\, a^4}_{\text{3 factors of } a^4} = a^{\underbrace{4+4+4}_{\text{Adding exponent 4 three times}}} = a^{12}$$

From arithmetic, we know that multiplication is repeated addition of the same number. Therefore adding the exponent 4 three times is the same as $3 \cdot 4$. Thus

Power of a power — Multiply exponents

$$(a^4)^3 = a^{4 \cdot 3} = a^{12}$$

Power of a power property of exponents

For all real numbers a and positive integers m and n,

$$(a^m)^n = a^{mn}$$

Concept

A power of a power is found by multiplying the exponents.

Example 3–1 C

Perform the indicated operations.

1. $(x^3)^5 = x^{3 \cdot 5} = x^{15}$

2. $(2^3)^4 = 2^{3 \cdot 4} = 2^{12} = 4{,}096$

3. $(a^{2n})^{3n} = a^{2n \cdot 3n}$ Power of a power, multiply the exponents
$= a^{6n^2}$

Group of factors to a power property of exponents

A third property of exponents can be derived using the definition of exponents and the commutative and associative properties of multiplication. Observe that

$$\begin{aligned}(ab)^3 &= \underbrace{ab \cdot ab \cdot ab}_{\text{3 factors of } ab}\\ &= \underbrace{a \cdot a \cdot a}_{\text{3 factors of } a} \cdot \underbrace{b \cdot b \cdot b}_{\text{3 factors of } b}\\ &= a^3b^3\end{aligned}$$

Group of factors to a power property of exponents

For all real numbers a and b and positive integers n,

$$(ab)^n = a^nb^n$$

Concept

When a group of factors is raised to a power, we will raise each of the factors in the group to this power.

■ Example 3–1 D

Perform the indicated operations.

1. $(xy)^5 = x^5y^5$

Group of factors to a power	Raise each factor to the power	Standard form
2. $(2ab)^4$	$= 2^4a^4b^4$	$= 16a^4b^4$

Note A common error is to forget to raise the numerical coefficient to the appropriate power. In example 2, 2 is raised to the fourth power.

3. $(2 + 3)^3 = (5)^3 = 125$

Note The quantity $(2 + 3)^3 \neq 2^3 + 3^3$ because 2 and 3 are *terms*, not factors, as the property requires. If we consider $(2 + 3)$ to be a single number, then by the definition of exponents we have

$$(2 + 3)^3 = (2 + 3)(2 + 3)(2 + 3)$$

or, in general,

$$(a + b)^3 = (a + b)(a + b)(a + b)$$

We will discuss the method of multiplying this product in section 3–3. ■

The following examples illustrate some problems in which more than one property is used within the problem.

■ **Example 3–1 E**

Perform the indicated operations.

1. $(2x^3y^4)^3$
$= 2^3(x^3)^3(y^4)^3$ — Each factor in the group is raised to the third power
$= 8x^9y^{12}$ — $2^3 = 8$ and power of a power, multiply exponents

2. $(3a^4b^6)^2$
$= 3^2(a^4)^2(b^6)^2$ — Each factor in the group is raised to the second power
$= 9a^8b^{12}$ — $3^2 = 9$ and power of a power, multiply exponents

3. $(-3a^2)(2ab^3)(-4a^3b^5)$
$= [(-3)(2)(-4)]\,(a^2aa^3)(b^3b^5)$ — Multiply like bases using the commutative and associative properties
$= 24a^6b^8$

4. $(3x^2y^5)^2(-2x^4y)^3$
$= 3^2(x^2)^2(y^5)^2(-2)^3(x^4)^3y^3$ — Each factor in each group is raised to the power outside
$= 9x^4y^{10} \cdot (-8)x^{12}y^3$ — Standard form for numbers, power of a power for variables
$= [9 \cdot (-8)](x^4x^{12})(y^{10}y^3)$ — Multiply like bases
$= -72x^{16}y^{13}$

5. $(3x^2)^2x^3 + (2x^3)^2x$
$= 3^2(x^2)^2x^3 + 2^2(x^3)^2x$ — Group of factors to a power
$= 9x^4x^3 + 4x^6x$ — Power of a power
$= 9x^7 + 4x^7$ — Multiplication of like bases, add exponents
$= 13x^7$ — Add like terms

6. $(5a^3)^2a^2 + (3a^4)^2a$
$= 5^2(a^3)^2a^2 + 3^2(a^4)^2a$ — Group of factors to a power
$= 25a^6a^2 + 9a^8a$ — Power of a power
$= 25a^8 + 9a^9$ — Multiplication of like bases, add exponents

Note In example 5, we were able to carry out the addition because we had like terms. In example 6, the addition was not performed because a^8 and a^9 are not like terms.

▶ ***Quick check*** Perform the indicated operations. $(5x^4y)^2$ ■

Mastery points

Can you

- Write a product in exponential form?
- Use the product property of exponents?
- Raise a power to a power?
- Raise a group of factors to a power?

Exercise 3–1

Write each expression in exponential form and identify the base and the exponent. See example 3–1 A.

Example $(-5)\cdot(-5)\cdot(-5)$

Solution $=(-5)^3$ Base -5, exponent 3

1. $(-2)(-2)(-2)(-2)$
2. $5\cdot5\cdot5\cdot5\cdot5\cdot5$
3. $x\cdot x\cdot x\cdot x\cdot x$
4. $y\cdot y\cdot y$
5. $(2x)(2x)(2x)(2x)$
6. $(ab^2)(ab^2)(ab^2)$
7. $(x^2+3y)(x^2+3y)(x^2+3y)$
8. $(2a^2-b)(2a^2-b)$
9. $-(2\cdot 2)$
10. $-(2\cdot2\cdot2\cdot2)$

Perform the indicated operations. See examples 3–1 B, C, D, and E.

Examples $a^2\cdot a\cdot a^5$ $\qquad$ $(5x^4y)^2$

Solutions $=a^{2+1+5}$ Multiply like bases $\qquad$ $=5^2(x^4)^2y^2$ Each factor in the group is raised to the second power

$=a^8$ Add exponents $\qquad$ $=25x^8y^2$ $5^2=25$ and power of a power, multiply exponents

11. $a^5\cdot a^4$
12. $x^3\cdot x^9$
13. $y\cdot y^2$
14. $b\cdot b^4$
15. $(-2)^3(-2)^3$
16. $(-3)(-3)^3$
17. $(-2)(-2)^3$
$\boxed{18}$. $(-2)(-2^2)$
19. $(-2^2)(3^2)$
20. $(-3^2)(2^2)$
21. $x^2\cdot x\cdot x^5$
22. $y^4\cdot y^2\cdot y$
23. $(x^2y^2)(x^5y^3)$
24. $(a^5b)(a^2b^3)$
25. $(2ab^2)(3a^2)$
26. $(-2a^2)(4a^5)$
27. $(-3x^3)(-2x^2)$
28. $(5ab^2)(2a^5b^4)$
29. $(8x^2y^5)(3xy^4)$
30. $(-6a^3b^7)(4ab^4)$
31. $(2^2)^3$
$\boxed{32}$. $(-2^2)^3$
33. $(-3^2)^3$
34. $(-3^3)^2$
35. $(-2^3)^2$
36. $(x^4)^7$
37. $(a^2)^6$
38. $(a^3b^6)^4$
39. $(x^2yz^3)^5$
40. $(5R^2S^5)^2$
41. $(-7s^4t^2)^2$
42. $(-3a^9b^7)^3$
43. $(-x^9y^{12}z^8)^4$
$\boxed{44}$. $(3x^2y)^2(2xy^3)$
45. $(a^2b^3)^4(ab^5)$
46. $(2a^3b^2)(a^5b^3)^4$
47. $(x^2y^6)^2(x^3y^4)^3$
48. $(-a^2b)(a^5b^2)^3$
49. $(x^4y^5)^2(-x^2y^8)$
50. $(-2a^2b^4)^3(-3ab^5)^2$
51. $(-5x^4y^2)^2(-3x^2y^7)$
$\boxed{52}$. $(2a^2)^2a^3+(3a)^3a^4$
53. $(3x^3)^2x^3+2x^5(2x^2)^2$
54. $(5b^2)^22b^7-(3b^4)^25b^3$
55. $(4a^5)^22a^3-(3a^4)^34a$
56. $(-2x^2)^33x^4+(5x^5)^2$
$\boxed{57}$. $(3x^5)^2-(2x^2)^3x^2$
58. $(x^4)^2x^3+(x^3)^3x$
59. $(2a^3)^33a^4+(3a^5)^22a$
60. $(b^5)^34b^3-(2b^3)^4b^2$
61. $(x^4)^33x^5+(2x^2)^53x^4$
$\boxed{62}$. $x^{5n}\cdot x^{4n}$
63. $a^{2b}\cdot a^{7b}$
64. $x^{6n}\cdot x^n$
65. $a^{5b}\cdot a^{4b}$
$\boxed{66}$. $x^{2n+1}\cdot x^{n+4}$
67. $a^{2b+5}\cdot a^{3b-2}$
68. $R^{3S}\cdot R^{2S+3}$
69. $x^{y-4}\cdot x^{2y+9}$
$\boxed{70}$. $(a^{3n})^{4n}$
71. $(x^{3y})^{5y}$
72. $(R^{2S})^S$

Solve the following word problems.

73. The amount A accumulated in a savings account earning 12% interest per year compounded monthly is given by $A=P(1.01)^n$, where P represents the amount of deposit and n is the number of months the money is left on deposit. Find A, if $P=5{,}000$ and $n=12$.

74. Find A in exercise 73, if $P=2{,}000$ and $n=6$.

75. The amount A of a radioactive substance remaining after time t can be found using the formula $A=A_0(0.5)^{t/n}$, where A_0 represents the original amount of radioactive material and n is the half-life given in the same units of time as t. If the half-life of nitrogen 13 is 10 minutes, how much radioactive nitrogen will remain after 20 minutes if we start with 10 grams?

76. If the half-life of uranium 229 is 58 minutes, how much radioactive uranium will remain after 174 minutes if we start with 80 ounces? (Refer to exercise 75.)


Love The Taste.
Taste The Love.
At Culver's® we can't think of anything better than serving up our creamy frozen custard and delicious classics cooked fresh the minute you order them. Which is why when we bring them to your table, they're always accompanied by a warm smile and a friendly offer to see if there's anything else we can get for you. So come on into your neighborhood Culver's and see for yourself. You might just be in love by the time you leave.
Culver's
FROZEN CUSTARD
BUTTERBURGERS
®
© 2005 Culver Franchising System, Inc.
www.culvers.com

Review exercises

Perform the indicated multiplication. See section 1–2.

1. $4 \cdot (-6)$ 2. -3^2 3. $4 \cdot 6 \cdot 0 \cdot 3$ 4. $(-5)^2$

Perform the indicated addition and subtraction. See section 1–6.

5. $6ab + 3ab$ 6. $a^2 - 5a + 3a - 15$ 7. $x^2 - 3x + 3x - 9$ 8. $3x^2 - y^2 - 2x^2 + 3y^2$

3–2 ■ Products of polynomials

Extended distributive property

In chapter 1, we stated the distributive property as

$$a(b + c) = ab + ac$$

Many problems have more than two terms inside the grouping symbol. We will now extend the distributive property to more than two terms and will use subscripts to state the *extended distributive property of multiplication over addition.*

Extended distributive property

$$a(b_1 + b_2 + \cdots + b_n) = ab_1 + ab_2 + \cdots + ab_n$$

Concept

When we multiply a multinomial by a monomial, we multiply each term of the multinomial by the monomial.

■ Example 3–2 A

Perform the indicated multiplication.

1. $2x^3(3x^2 - 5x + 7)$

   We multiply the monomial $2x^3$ times each term in the trinomial to get

   $(2x^3)(3x^2) + (2x^3)(-5x) + (2x^3)(7)$

   In each indicated product, we multiply the coefficient and add the exponents of the like bases to get

   $2x^3(3x^2 - 5x + 7) = 6x^5 - 10x^4 + 14x^3$

2. $4a^2b^3(2a^2 - 3ab + 4b^2) = (4a^2b^3)(2a^2) + (4a^2b^3)(-3ab) + (4a^2b^3)(4b^2)$
   $= 8a^4b^3 - 12a^3b^4 + 16a^2b^5$

▶ ***Quick check*** Perform the indicated multiplication. $3b^2(2b^5 - b + 4)$ ■

Multiplication of multinomials

The product of two binomials will require the use of the distributive property several times. That is, in the product

$$(x + y)(2x + y)$$

we consider $(x + y)$ as a single factor and apply the distributive property.

$$(x + y)(2x + y) = (x + y) \cdot 2x + (x + y) \cdot y$$

We now apply the distributive property again.

$$\begin{aligned}(x + y)\cdot 2x + (x + y)\cdot y &= x\cdot 2x + y\cdot 2x + x\cdot y + y\cdot y\\ &= 2x^2 + 2xy + xy + y^2\end{aligned}$$

The last step in the problem is to combine like terms, if there are any.

$$2x^2 + (2xy + xy) + y^2 = 2x^2 + 3xy + y^2$$

Notice that in this product each term of the first factor is multiplied by each term of the second factor. We can generalize our procedure as follows:

Multiplying two multinomials

When multiplying two multinomials, we multiply each term of the first multinomial by each term of the second multinomial and then combine like terms.

■ Example 3–2 B

Perform the indicated multiplication and simplify.

1. $(a + 4)(a + 1)$

   When we perform the multiplication of two binomials, the four products that are obtained can be seen more clearly by using arrows to indicate the multiplication that is being carried out.

$$\begin{aligned}(a + 4)(a + 1) &= \overset{1}{a\cdot a} + \overset{2}{a\cdot 1} + \overset{3}{4\cdot a} + \overset{4}{4\cdot 1} && \text{Distribute multiplication}\\ &= a^2 + a + 4a + 4 && \text{Multiply the monomials}\\ &= a^2 + 5a + 4 && \text{Combine like terms}\end{aligned}$$

Note We have drawn arrows to indicate the multiplication that is being carried out. This is a convenient way for us to indicate the multiplication to be performed.

2. $$\begin{aligned}(2x + 3)(5x - 2) &= \overset{F}{10x^2} - \overset{O}{4x} + \overset{I}{15x} - \overset{L}{6} && \text{Distribute and multiply}\\ &= 10x^2 + 11x - 6 && \text{Combine like terms}\end{aligned}$$

Note A word that is useful for remembering the multiplication to be performed when multiplying two binomials is **FOIL.** Foil is an abbreviation signifying **F**irst times first, **O**uter times outer, **I**nner times inner, and **L**ast times last.

3. $$\begin{aligned}(3a - 2b)(2a - 5b) &= \overset{F}{6a^2} - \overset{O}{15ab} - \overset{I}{4ab} + \overset{L}{10b^2} && \text{Distribute and multiply}\\ &= 6a^2 - 19ab + 10b^2 && \text{Combine like terms}\end{aligned}$$

▶ ***Quick check*** Perform the indicated multiplication and simplify.
$(3x - 1)(2x + 3)$

In arithmetic, we multiply numbers stated vertically. We can use this same procedure to multiply two multinomials. To perform the example 3 multiplication vertically, we would proceed as follows:

$$\begin{array}{r} 2a - 5b \\ \underline{3a - 2b} \end{array}$$

Multiply $-2b$ and $2a - 5b$.

$$\begin{array}{r} 2a - 5b \\ \underline{3a - 2b} \\ -4ab + 10b^2 \end{array}$$

Multiply $3a$ and $2a - 5b$. Line up any like terms in the same columns.

$$\begin{array}{rrr} & 2a & - 5b \\ & \underline{3a} & \underline{- 2b} \\ & -4ab & + 10b^2 \\ \underline{6a^2} & \underline{- 15ab} & \\ \end{array}$$

Add like terms.

$$\begin{array}{rrr} & 2a & - 5b \\ & \underline{3a} & \underline{- 2b} \\ & -4ab & + 10b^2 \\ \underline{6a^2} & \underline{- 15ab} & \\ 6a^2 & - 19ab & + 10b^2 \end{array}$$

Special products

Three special products appear so often that we should be able to write the answer without computation. Consider the product

$$(a + b)^2 = (a + b)(a + b)$$

which becomes

$$a^2 + ab + ab + b^2$$

and combining like terms, we get

$$a^2 + 2ab + b^2$$

This is called the **square of a binomial** and has certain characteristics. Inspection shows us that in

$$(a + b)^2 = a^2 + 2ab + b^2$$

the three terms of the product can be obtained in the following manner:

Square of a binomial

1. The first term of the product is the *square of the first term* of the binomial $[(a)^2 = a^2]$.
2. The second term of the product is *two times the product of the two terms of the binomial* $[2(a \cdot b) = 2ab]$.
3. The third term of the product is the *square of the second term* of the binomial $[(b)^2 = b^2]$.

Our second special product is

$$(a - b)^2 = (a - b)(a - b)$$

which becomes

$$a^2 - ab - ab + b^2$$

and simplifies to

$$a^2 - 2ab + b^2$$

This also is called the square of a binomial. We can apply the previous procedure to this special product if we take the sign of the operation (+ or −) as the sign of the coefficient.

$$\begin{aligned}(a - b)^2 = [a + (-b)]^2 &= (a)^2 + 2[a \cdot (-b)] + (-b)^2 \\ &= a^2 - 2ab + b^2\end{aligned}$$

Square of a binomial

For real numbers a and b,

$$(a + b)^2 = a^2 + 2ab + b^2 \quad \text{and}$$
$$(a - b)^2 = a^2 - 2ab + b^2$$

Concept

(1st term + 2nd term)^2 =
(1st term)^2 + 2(1st term · 2nd term) + (2nd term)^2

■ Example 3–2 C

Perform the indicated multiplication and simplify.

1. $(x + 3)^2$

The first term of the binomial is x and the second term of the binomial is 3. Substituting into the special product property, we have

$$(x + 3)^2 = (\underset{\substack{\uparrow \\ \text{1st} \\ \text{term}}}{x})^2 + 2(\underset{\substack{\uparrow \\ \text{1st} \\ \text{term}}}{x} \cdot \underset{\substack{\uparrow \\ \text{2nd} \\ \text{term}}}{3}) + (\underset{\substack{\uparrow \\ \text{2nd} \\ \text{term}}}{3})^2 \qquad \text{Special product property}$$

$$= x^2 + 6x + 9 \qquad \text{Multiply monomials}$$

Note $(x + 3)^2 = x^2 + 6x + 9$, *not* $x^2 + 9$. This is a common error.

2. $(2a + b)^2 = (2a)^2 + 2(2a \cdot b) + (b)^2$ Special product property
$= 4a^2 + 4ab + b^2$ Multiply monomials

3. $(2x - y)^2$
The first term of the binomial is $2x$ and the second term of the binomial is $-y$. Substituting into the formula, we have

$$(2x - y)^2 = (\underset{\substack{\uparrow \\ \text{1st} \\ \text{term}}}{2x})^2 + 2[(\underset{\substack{\uparrow \\ \text{1st} \\ \text{term}}}{2x})(\underset{\substack{\uparrow \\ \text{2nd} \\ \text{term}}}{-y})] + (\underset{\substack{\uparrow \\ \text{2nd} \\ \text{term}}}{-y})^2 \quad \text{Special product property}$$
$$= 4x^2 - 4xy + y^2 \quad \text{Multiply monomials}$$

4. $(4a - 3b)^2 = (4a)^2 + 2[(4a)(-3b)] + (-3b)^2$ Special product property
$= 16a^2 - 24ab + 9b^2$ Multiply monomials

▶ ***Quick check*** Perform the indicated multiplication and simplify. $(x + 3y)^2$ ■

The third special product is obtained by multiplying the sum and difference of the same two terms. Consider the product

$$(a + b)(a - b) = a^2 - ab + ab - b^2$$
$$= a^2 - b^2$$

This special product is called the **difference of two squares.**

Difference of two squares

In general, for real numbers a and b,

$$(a + b)(a - b) = a^2 - b^2$$

Concept

The product is obtained by first squaring the first term of the factors and then subtracting the square of the second term of the factors.

(1st term + 2nd term)(1st term − 2nd term) =
(1st term)^2 − (2nd term)^2

Note We can consider b or $-b$ the second term since the square of each is b^2. That is,

$$(b)^2 = (-b)^2 = b^2$$

■ **Example 3–2 D**

Perform the indicated multiplication and simplify.

1. $(x + 5)(x - 5)$
The first term is x and the second term is 5. Substituting into the special product property, we have

$$(x + 5)(x - 5) = (\underset{\substack{\uparrow \\ \text{1st} \\ \text{term}}}{x})^2 - (\underset{\substack{\uparrow \\ \text{2nd} \\ \text{term}}}{5})^2 \quad \text{Special product property}$$
$$= x^2 - 25 \quad \text{Standard form}$$

2. $(2a + b)(2a - b) = (2a)^2 - (b)^2$ Special product property
$= 4a^2 - b^2$ Standard form ■

In all the examples we have considered, whether they were special products or not, a single procedure is sufficient. When multiplying two multinomials, **we multiply each term of the first multinomial by each term of the second multinomial and then combine like terms.**

Example 3–2 E

Perform the indicated operations and simplify.

1. $(a + 3)(2a^2 - a + 4)$

$= (a)(2a^2) + (a)(-a) + (a)(4) + (3)(2a^2) + (3)(-a) + (3)(4)$	Distribute multiplication
$= 2a^3 - a^2 + 4a + 6a^2 - 3a + 12$	Multiply monomials
$= 2a^3 + 5a^2 + a + 12$	Combine like terms

2. $(x + y)(x + 2y)(x - y)$

When there are three quantities to be multiplied, we apply the associative property to multiply two of them together first and take that product times the third.

$[(x + y)(x + 2y)](x - y)$

$= [x^2 + 2xy + xy + 2y^2](x - y)$	Multiply the first two groups
$= [x^2 + 3xy + 2y^2](x - y)$	Combine like terms in brackets
$= x^3 - x^2y + 3x^2y - 3xy^2 + 2xy^2 - 2y^3$	Distribute multiplication
$= x^3 + 2x^2y - xy^2 - 2y^3$	Combine like terms

Alternate If we had observed in example 2 that the first and last quantities were a special product, we could have used the commutative and associative properties to multiply them together first.

$[(x + y)(x - y)](x + 2y)$

$= [x^2 - y^2](x + 2y)$	Multiply the first two groups (special product)
$= x^3 + 2x^2y - xy^2 - 2y^3$	Distribute multiplication

3. $(a + 2b)^3$

$= (a + 2b)(a + 2b)(a + 2b)$	Write in expanded form
$= [(a + 2b)(a + 2b)](a + 2b)$	Multiply the first two groups
$= [(a)^2 + 2(a)(2b) + (2b)^2](a + 2b)$	Special product, square of a binomial
$= [a^2 + 4ab + 4b^2](a + 2b)$	Simplify within brackets
$= a^3 + 2a^2b + 4a^2b + 8ab^2 + 4ab^2 + 8b^3$	Distribute multiplication
$= a^3 + 6a^2b + 12ab^2 + 8b^3$	Combine like terms

4. $(2a - 5)^2 - (a + 3)(a - 4)$

We must take care of all multiplication before we can perform the indicated subtraction.

$(2a - 5)^2 - (a + 3)(a - 4)$

$= [(2a)^2 + 2(2a)(-5) + (-5)^2] - [a^2 - 4a + 3a - 12]$	
$= [4a^2 - 20a + 25] - [a^2 - a - 12]$	
$= 4a^2 - 20a + 25 - a^2 + a + 12$	Remove brackets
$= 3a^2 - 19a + 37$	Combine like terms

Note In example 4, it is necessary to place grouping symbols around the products since the order of operations requires that multiplication be done before subtraction. In the second line, if we did not use parentheses around the product, the line would be $4a^2 - 20a + 25 - a^2 - a - 12$, and we would have subtracted only the first term of the product and not the entire product as was indicated.

5. $-2\{3x + 2[x - (3x + 1)]\}$

$= -2\{3x + 2[x - 3x - 1]\}$	Remove parentheses
$= -2\{3x + 2[-2x - 1]\}$	Combine like terms
$= -2\{3x - 4x - 2\}$	Distributive property
$= -2\{-x - 2\}$	Combine like terms
$= 2x + 4$	Distributive property

Mastery points

Can you

- Multiply a monomial by a multinomial?
- Multiply two multinomials?
- Multiply two multinomials vertically?
- Find the square of a binomial and the difference of two squares using the special products?

Exercise 3–2

Perform the indicated multiplication and simplify. See example 3–2 A.

Example $3b^2(2b^5 - b + 4)$

Solution $= (3b^2)(2b^5) + (3b^2)(-b) + (3b^2)(4)$ Distribute $3b^2$ times each term in parentheses
$= 6b^7 - 3b^3 + 12b^2$ Multiply monomials

1. $a(2a^2 - 3a + 4)$
2. $2x(5x^2 - 3x + 7)$
3. $-2y(3y^2 - 5y + 4)$
4. $-4x(2x^2 - 3x - 9)$
5. $3a^2(4a^2 - 2ab + 3b^2)$
6. $5x^3(2x^2 - 3x + 4)$
7. $6xy(5x^2y - 4xy^3 + 2xy)$
8. $-a^3b(3a^2b^5 - ab^4 - 7a^2b)$
9. $-5x^3y^4(2x^2y + 7xy^6 - 3x^2y^5)$
10. $6a^3b^5(-4a^2b + 5a^3b^3 - 8a^5b + 2)$

Perform the indicated multiplication and simplify (a) by multiplying horizontally and (b) by multiplying vertically. See example 3–2 B.

Example $(3x - 1)(2x + 3)$

Solution $= 6x^2 + 9x - 2x - 3$ Distribute multiplication
$= 6x^2 + 7x - 3$ Combine like terms

11. $(a + 5)(a + 3)$
12. $(x + 4)(x + 6)$
13. $(b + 4)(b - 5)$
14. $(x + 7)(x - 8)$
15. $(2x - y)(x + y)$
16. $(3a + b)(a + b)$
17. $(5x - y)(2x + y)$
18. $(3x + y)(4x - y)$
19. $(7x - 5y)(6x + 4y)$
20. $(4a - 7b)(3a - 5b)$

Perform the indicated multiplication and simplify. Use any of the special products where possible. See examples 3–2 C, D, and E.

Example $(x + 3y)^2$

Solution $= (x)^2 + 2[(x)(3y)] + (3y)^2$ Special products, square of a binomial
$= x^2 + 6xy + 9y^2$ Standard form

21. $(x + 3)^2$
22. $(2a + b)^2$
23. $(3x + y)^2$
24. $(5x + 2y)^2$
25. $(4x + 3y)^2$
26. $(2x - 3)^2$
27. $(2a - 5)^2$
28. $(3a - b)^2$
29. $(4x - 3y)^2$
30. $(a - 3b)(a + 3b)$
31. $(x - 3y)(x + 3y)$
32. $(2x - 3yz)(2x + 3yz)$
33. $(5a + 2bc)(5a - 2bc)$
34. $(a + 2b)(a^2 - ab + b^2)$
35. $(x + y)(x^2 - 3xy + 2y^2)$
36. $(x - 2y)(3x^2 + 4xy - 2y^2)$
37. $(3a - 2b)(5a^2 - 7ab + 3b^2)$
38. $(x^2 - 2x + 1)(x^2 + 3x + 2)$
39. $(x^2 + 3x + 4)(x^2 - 2x - 3)$
40. $(2a^2 - 3a + 5)(a^2 - 4a + 2)$
41. $(5b^2 - 3b + 7)(b^2 + b + 3)$
42. $(a - 3b)^3$
43. $(x + 2y)^3$
44. $(2x - 3y)^3$
45. $(4a - b)^3$
46. $(x - 2y)(2x + y)(x - y)$
47. $(a - 2b)(2a - b)(a + 2b)$

Perform the indicated operations, remove all grouping symbols, and simplify. See example 3–2 E.

48. $(x + 2)^2 + (x - 4)^2$
49. $(a + 5)^2 + (a - 2)^2$
50. $(2x - 1)(x + 3) - (x + 2)^2$
51. $(3x - 2)(x + 5) - (x - 4)^2$
52. $(2x + 3)^2 - (3x + 1)(x - 2)$
53. $(y + 3)(2y - 4) - (y + 5)(y - 4)$
54. $(a + 3)(a - 4) - (3a - 2)^2$
55. $3[b - (3b + 2) + 5]$
56. $-2[5a - (2a + 3) - 3(3a + 7)]$
57. $2x[3x - 2(5x + 1)]$
58. $3y[-2y + 3(5y - 4)]$
59. $-2\{3x - 2[5x - (7 - 3x)]\}$
60. $-5x\{2x - 4[x + (3x - 1)]\}$

Perform the indicated multiplication and simplify. Assume that all variables used as exponents represent positive integers.

Examples $x^2(x^n + 1)$

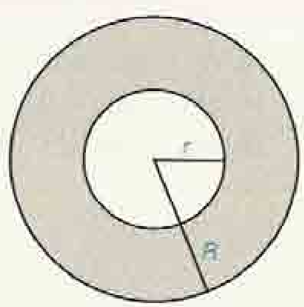
Solutions $= x^2 \cdot x^n + x^2 \cdot 1$
$= x^{n+2} + x^2$

$(x^n - 1)(x^n + 3)$
$= x^n \cdot x^n + x^n \cdot 3 + (-1) \cdot x^n + (-1) \cdot 3$
$= x^{n+n} + 3x^n - x^n - 3$
$= x^{2n} + 2x^n - 3$

61. $a^n(a^2 + 3)$
62. $b^{2n}(b^3 - 1)$
63. $x^n(3x^n + 1)$
64. $x^{n+2}(x^{n+1} + x)$
65. $x^{n-2}(x^{n+5} - x^2)$
66. $(a^n + 1)(a^n - 2)$
67. $(b^n - 3)(b^n - 2)$
68. $(2a^n - b^n)^2$
69. $(3x^n + y^n)^2$
70. $(x^{2n} + y^n)(x^{2n} - y^n)$
71. $(a^n + b^{3n})(a^n - b^{3n})$
72. $(3x^{2n} - 2y^{3n})^2$

Solve the following word problems.

73. The area of the shaded region between two circles is $\pi(R + r)(R - r)$. Perform the indicated multiplication. (π is the lowercase Greek letter pi.)



74. When squares of c units on a side are cut from the corners of a square sheet of metal x units on a side and folded up into a tray, its volume is $c(x - 2c)(x - 2c)$ cubic units. Perform the indicated multiplication.

75. The total area of the surface of a cylinder is determined by $A = 2\pi r(h + r)$. Multiply in the right member.

76. The equation for the distance traveled by a rocket fired vertically upward into the air is given by $S = 16t(35 - t)$, where the rocket is S feet from the ground after t seconds. Multiply in the right member.

77. In engineering, the equation for the deflection of a beam is given by

$$Y = \frac{Wx}{48EI}(2x^3 - 3\ell x^2 - \ell^3)$$

Multiply in the right member.

78. In engineering, the equation of transverse shearing stress in a rectangular beam is given in two forms:

(a) $T = \frac{V}{8I}(h + 2V_1)(h - 2V_1)$ and

(b) $T = \frac{3V}{2A}\left(1 + \frac{2V_1}{H}\right)\left(1 - \frac{2V_1}{H}\right)$

Multiply in the right member of each equation.

Review exercises

Perform the indicated operations. See section 1–2.

1. $(-4) + (-6)$
2. $(-3)(-7)$
3. $(-4) - (-8)$
4. -4^2

Simplify by using the properties of exponents. See section 3–1.

5. $a^3 \cdot a^5$
6. $(x^3)^4$
7. $x \cdot x^2 \cdot x^3$
8. $(2ab^3)^2$

3–3 ■ Further properties of exponents

Quotient property of exponents

Another useful property of exponents can be seen from the following example. Consider the expression

$$\frac{a^5}{a^3}, a \neq 0$$

We use the definition of exponents to write the fraction as

$$\frac{a^5}{a^3} = \frac{a \cdot a \cdot a \cdot a \cdot a}{a \cdot a \cdot a}$$

We reduce the fraction and get

$$\frac{\cancel{a} \cdot \cancel{a} \cdot \cancel{a} \cdot a \cdot a}{\cancel{a} \cdot \cancel{a} \cdot \cancel{a}} = \frac{a \cdot a}{1} = a \cdot a = a^2$$

We reduced by three factors of a, leaving $5 - 3 = 2$ factors of a in the numerator. Therefore

$$\frac{a^5}{a^3} = a^{5-3} = a^2$$

Quotient property of exponents

For all real numbers a, $a \neq 0$, and integers m and n,

$$a^n \div a^m = \frac{a^n}{a^m} = a^{n-m}$$

Concept

To divide quantities having **like bases,** subtract the exponent of the denominator from the exponent of the numerator to get the exponent of the common base in the quotient.

Note If $a = 0$, we have an expression that has no meaning. Therefore $a \neq 0$ indicates that we want our variable to assume no value that would cause the denominator to be zero.

Example 3–3 A

Perform the indicated operations and simplify. Assume that no variable is equal to zero.

1. $\dfrac{a^9}{a^6} = a^{9-6} = a^3$

2. $b^{12} \div b^4 = b^{12-4} = b^8$

3. $\dfrac{2^8}{2^3} = 2^{8-3} = 2^5 = 32$ — When dividing numbers with like bases raised to a power, the division is carried out by means of subtracting exponents. *The base is unaltered.*

4. $\dfrac{3^3x^{12}y^9}{3x^4y^8} = 3^{3-1} \cdot x^{12-4} \cdot y^{9-8}$ — Division of like bases

   $= 3^2x^8y^1$ — Subtract exponents

   $= 9x^8y$ — Standard form

Negative exponents

Until now, we have considered only those problems in which the exponent of the numerator is greater than the exponent of the denominator. Consider the example

$$\frac{a^3}{a^5}, a \neq 0$$

By the definition of exponents, this becomes

$$\frac{a^3}{a^5} = \frac{a \cdot a \cdot a}{a \cdot a \cdot a \cdot a \cdot a}$$

and reducing the fraction,

$$\frac{a \cdot a \cdot a}{a \cdot a \cdot a \cdot a \cdot a} = \frac{1}{a \cdot a} = \frac{1}{a^2}$$

Again we reduced by three factors of a, leaving $5 - 3 = 2$ factors of a in the denominator. Hence

$$\frac{a^3}{a^5} = \frac{1}{a^2}$$

However if we use the quotient property of exponents to carry out the division,

$$\frac{a^3}{a^5} = a^{3-5} = a^{-2}$$

Since we should arrive at the same answer regardless of which procedure we use, then a^{-2} must be equivalent to $\frac{1}{a^2}$, that is, $a^{-2} = \frac{1}{a^2}$. This leads us to the definition of a^{-n}.

Definition of negative exponents

For all real numbers a, $a \neq 0$, and positive integers n,

$$a^{-n} = \frac{1}{a^n}$$

Concept

When a symbol is raised to a negative exponent, we can rewrite it as the reciprocal of that symbol to the positive exponent.

■ Example 3–3 B

Write the following without negative exponents. Assume that no variable is equal to zero.

1. $3^{-2} = \frac{1}{3^2}$ Rewrite as 1 over 3 to the positive two

$= \frac{1}{9}$ Standard form

Note In example 1, the value of 3^{-2} cannot be determined until the exponent is made positive.

2. $a^{-4} = \frac{1}{a^4}$

Note From the definition of negative exponents, if a factor is moved either from the numerator to the denominator or from the denominator to the numerator, the sign of its exponent will change. The sign of the base will not be affected by this change.

Alternative procedure:

$a^{-4} = \frac{a^{-4}}{1}$ Rewrite as a fraction

$= \frac{1}{a^4}$ The sign of the exponent is changed as the factor is moved from the numerator to the denominator

3. $\frac{1}{x^{-4}} = \frac{x^4}{1}$ The sign of the exponent is changed as the factor is moved from the denominator to the numerator

$= x^4$ Standard form

4. $-b^{-3} = -(b^{-3}) = -\left(\frac{1}{b^3}\right) = -\frac{1}{b^3}$

Note Only the sign of the exponent changes, not the sign of the base. ■

Zero as an exponent

We now examine the situation involving the division of like bases raised to the same power. For example, consider

$$\frac{a^3}{a^3}, a \neq 0$$

By the definition of exponents, we have

$$\frac{a^3}{a^3} = \frac{a \cdot a \cdot a}{a \cdot a \cdot a} = \frac{1}{1} = 1$$

and by the quotient property of exponents,

$$\frac{a^3}{a^3} = a^{3-3} = a^0$$

Since

$$\frac{a^3}{a^3} = 1 \text{ and } \frac{a^3}{a^3} = a^0$$

then for consistency, we make the following definition for a^0.

Definition of zero as an exponent

For all real numbers a, $a \neq 0$,

$$a^0 = 1$$

Concept

Any number other than zero raised to the zero power is equal to 1.

Note All the properties of exponents stated so far now apply for any integer used as an exponent.

Example 3–3 C

Write the following expressions without using zero as an exponent. Assume that no variable is equal to zero.

1. $x^0 = 1$
2. $8^0 = 1$
3. $(-5)^0 = 1$
4. $4x^0 = 4 \cdot x^0$ Only x is raised to the 0 power
   $= 4 \cdot 1$ x^0 is 1
   $= 4$ Standard form

Note The exponent acts only on the symbol immediately to its left. In this example, only x is raised to the zero power. The exponent of 4 is understood to be 1.

5. $-5^0 = -(5^0)$ This is not the same as $(-5)^0$
   $= -(1)$ 5^0 is 1
   $= -1$ Standard form

► ***Quick check*** Write $3b^0$ without using zero as an exponent.

Example 3–3 D

Perform the indicated operations. Leave the answer with only positive exponents. Assume that no variable is equal to zero.

1. $\dfrac{a^3b^2c^4}{ab^5c^4} = a^{3-1}b^{2-5}c^{4-4} = a^2b^{-3}c^0 = a^2 \cdot \dfrac{1}{b^3} \cdot 1 = \dfrac{a^2}{b^3}$

2. $\dfrac{x^3y^4}{x^5y^9} = x^{3-5} \cdot y^{4-9} = x^{-2}y^{-5} = \dfrac{1}{x^2y^5}$

3. $b^5 \cdot b^{-3} = b^{5+(-3)} = b^2$

4. $\dfrac{x^{-3}y^4}{x^{-7}y^9} = x^{(-3)-(-7)}y^{4-9} = x^4y^{-5} = \dfrac{x^4}{y^5}$

5. $(3a^5)^{-2} = \dfrac{1}{(3a^5)^2} = \dfrac{1}{3^2(a^5)^2} = \dfrac{1}{9a^{10}}$

Alternative solution:

$$(3a^5)^{-2} = 3^{-2}(a^5)^{-2} = 3^{-2} \cdot a^{-10} = \frac{1}{3^2a^{10}} = \frac{1}{9a^{10}}$$

In example 5, we see that there can be alternate methods of working problems involving exponents, depending on the order in which the properties are applied.

▶ ***Quick check*** Perform the indicated operations for $\dfrac{a^{-2}b^3}{a^{-4}b^6}$, leaving the answer with only positive exponents. Assume that no variable is equal to zero. ■

Fraction to a power property of exponents

Our last property of exponents can be derived from the definition of exponents. Consider the expression $\left(\dfrac{a}{b}\right)^3$, $b \neq 0$.

$$\left(\frac{a}{b}\right)^3 = \underbrace{\frac{a}{b} \cdot \frac{a}{b} \cdot \frac{a}{b}}_{3 \text{ factors of } \frac{a}{b}} = \frac{\overbrace{a \cdot a \cdot a}^{3 \text{ factors of } a}}{\underbrace{b \cdot b \cdot b}_{3 \text{ factors of } b}} = \frac{a^3}{b^3}$$

Thus

Fraction raised to a power → $\left(\dfrac{a}{b}\right)^3 = \dfrac{a^3}{b^3}$ ← Numerator raised to the power; ← Denominator raised to the power

Fraction to a power property of exponents

For all real numbers a and b, $b \neq 0$, and integers n,

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

Concept

Whenever a fraction is raised to a power, we raise the numerator to that power and place it over the denominator raised to that power.

Example 3–3 E

Perform the indicated operations and simplify. Leave the answer with only positive exponents. Assume that no variable is equal to zero.

1. $\left(\frac{2x}{y}\right)^3 = \frac{(2x)^3}{y^3}$ Both numerator and denominator are raised to the third power

$= \frac{2^3x^3}{y^3}$ Each factor in the numerator is raised to the third power

$= \frac{8x^3}{y^3}$ 2^3 in standard form is 8

2. $\left(\frac{x^{-3}y}{z^4}\right)^{-2} = \frac{(x^{-3}y)^{-2}}{(z^4)^{-2}}$ Numerator and denominator are raised to the power

$= \frac{(x^{-3})^{-2}y^{-2}}{(z^4)^{-2}}$ Numerator has a group of factors to a power

$= \frac{x^{(-3)(-2)}y^{-2}}{z^{(4)(-2)}}$ Power of a power

$= \frac{x^6y^{-2}}{z^{-8}}$ Multiply exponents

$= \frac{x^6z^8}{y^2}$ Standard form, factors raised to a negative power are moved to the other side of the fraction bar

3. $\left(\frac{3x}{y^3}\right)^2\left(\frac{y^4}{x^3}\right)^3 = \frac{(3x)^2}{(y^3)^2}\cdot\frac{(y^4)^3}{(x^3)^3}$ Numerators and denominators are raised to the power

$= \frac{3^2x^2}{y^6}\cdot\frac{y^{12}}{x^9}$ Group of factors to a power and power of a power

$= \frac{9x^2y^{12}}{x^9y^6}$ Multiply fractions

$= 9\cdot x^{2-9}\cdot y^{12-6}$ Division with like bases

$= 9x^{-7}y^6$ Subtract exponents

$= \frac{9y^6}{x^7}$ Standard form ■

Scientific notation

An important use of integer exponents is in scientific areas where we deal with very large or very small numbers. For example, the mass of a hydrogen atom is 0.000 000 000 000 000 000 000 001 67 gram; the mass of an electron is 0.000 000 000 000 000 000 000 000 000 91 gram; the half-life of lead 204 is 14,000,000,000,000,000,000 years.

Working with such numbers becomes quite difficult. Therefore, we convert such numbers into a more manageable form called **scientific notation.** We define the scientific notation form of a positive number Y to be the product

$$Y = a \times 10^n$$

where a is a number greater than or equal to 1 and less than 10, and n is an integer. Use the following steps to achieve this form of the number Y.

Scientific notation

Step 1 Move the decimal point to a position immediately following the first nonzero digit in *Y*.

Step 2 Count the number of places the decimal point has been moved. This is the exponent, *n*, to which 10 is raised.

Step 3 If
a. the decimal point is moved to the *left*, *n* is *positive*.
b. the decimal point is moved to the *right*, *n* is *negative*.
c. the decimal point already follows the first nonzero digit, *n* is *zero*.

Example 3–3 F

Express the following in scientific notation.

1. $9{,}000{,}000 = 9.000000. \times 10^6 = 9 \times 10^6$
2. $0.00000467 = 0.000004.67 \times 10^{-6} = 4.67 \times 10^{-6}$
3. $4.37 = 4.37 \times 10^0$
4. $-0.00341 = -0.003.41 \times 10^{-3} = -3.41 \times 10^{-3}$

Standard form

Sometimes it is necessary to convert a number in scientific notation to its standard form. To do this, we apply the procedure in reverse.

Standard form

When the exponent in 10^n is
1. *positive*, the decimal point is moved to the *right n* places.
2. *negative*, the decimal point is moved to the *left n* places.
3. *zero*, the decimal point is not moved.

Example 3–3 G

Express the following in standard form.

1. $6.37 \times 10^4 = 6.3700. = 63{,}700$
2. $4.81 \times 10^{-5} = 0.00004.81 = 0.0000481$
3. $8.59 \times 10^0 = 8.59$
4. $-3.48 \times 10^{-1} = -0.3.48 = -0.348$

Scientific notation can be used to simplify numerical calculations when the numbers are very large or very small. We first change the numbers to scientific notation and then use the properties of exponents to help perform the indicated operations.

■ Example 3–3 H

Perform the indicated operations using scientific notation.

1. $(198{,}000{,}000)(0.00347)$

$= (1.98 \times 10^8)(3.47 \times 10^{-3})$	Scientific notation
$= (1.98 \cdot 3.47) \times (10^8 \cdot 10^{-3})$	Commutative and associative properties
$= 6.8706 \times 10^5$	Multiply
$= 687{,}060$	Standard form

2. $\dfrac{(92{,}000{,}000)(0.0036)}{(0.018)(4{,}000)}$

$= \dfrac{(9.2 \times 10^7)(3.6 \times 10^{-3})}{(1.8 \times 10^{-2})(4.0 \times 10^3)}$	Scientific notation
$= \dfrac{(9.2)(3.6)10^7 \cdot 10^{-3}}{(1.8)(4.0)10^{-2} \cdot 10^3}$	Commutative and associative properties
$= \dfrac{(9.2)(3.6)}{(1.8)(4.0)} \cdot \dfrac{10^4}{10^1}$	Properties of exponents
$= 4.6 \times 10^3$	Division
$= 4{,}600$	Standard form

▶ ***Quick check*** Multiply $(473)(0.0000579)$ using scientific notation. ■

Mastery points

Can you

- Perform division on factors involving exponents?
- Perform operations involving negative exponents?
- Perform operations involving zero as an exponent?
- Raise a fraction to a power?
- Use scientific notation?

Exercise 3–3

Assume that all variables or groupings in this exercise represent nonzero real numbers and that all variables used as exponents represent positive integers.

Write each answer with only positive exponents. See examples 3–3 B and C.

Example $3b^0$

Solution

$= 3 \cdot b^0$	Only b is raised to the zero power
$= 3 \cdot 1$	b^0 is 1
$= 3$	Standard form

1. 5^0 **2.** $(-3)^0$ **3.** $(2a^3 - b)^0$ **4.** $(5x^2 + 4y)^0$ **5.** $7x^0$

6. $9y^0$ **7.** a^{-4} **8.** b^{-5} $\boxed{\textbf{9.}}$ $3a^{-2}$ **10.** x^2y^{-5}

11. $ab^{-4}c^{-3}$ $\boxed{\textbf{12.}}$ $\dfrac{1}{4a^{-3}}$ **13.** $\dfrac{1}{2x^{-5}}$ **14.** $\dfrac{1}{x^2y^{-4}}$

Perform all indicated operations and leave the answer with only positive exponents. See examples 3–3 A, D, and E.

Example $\dfrac{a^{-2}b^3}{a^{-4}b^6}$

Solution
$= a^{(-2)-(-4)}b^{3-6}$ Division of like bases
$= a^2b^{-3}$ Subtract exponents
$= \dfrac{a^2}{b^3}$ Standard form

15. $\dfrac{a^3}{a^4a^7}$ 16. $\dfrac{x^5y^3}{xy^5}$ 17. $\dfrac{2^3x^4y^9}{2xy^3}$ 18. $\dfrac{3^3a^4b^5}{3^2a^2b^3}$ 19. $\dfrac{3x^2y^5}{3^4x^5y^5}$

20. $\dfrac{5^2a^4b}{5^3a^9b^4}$ 21. 2^{-2} $\boxed{22.}$ -2^{-2} 23. -3^{-4} 24. $(-2)^{-3}$

25. $\dfrac{2^{-4}}{2^{-2}}$ 26. $\dfrac{3^{-6}}{3^{-4}}$ 27. $x^{-5}x^2$ $\boxed{28.}$ $(2a^{-3})(3a^{-5})$ 29. $(5x^2)(4x^{-7})$

30. $(3x^{-4}y^3)(2x^2y^{-1})$ 31. $(5a^{-3}b^{-4})(2a^2b^5)$ 32. $(2a^3)^{-3}$ 33. $(3x^4)^{-2}$

34. $(5a^2b^{-3})^{-2}$ 35. $(4a^{-3}b^4)^{-3}$ 36. $(x^{-3}y^0)^2$ 37. $(2a^{-2}b^0)^3$

38. $(5x^{-4}y^0z^2)^{-2}$ 39. $(3a^2b^{-3}c^0)^{-3}$ 40. $\left(\dfrac{2x}{y^3}\right)^2$ 41. $\left(\dfrac{3a^2}{b}\right)^3$

42. $\left(\dfrac{5a^2b}{c^4}\right)^2$ 43. $\left(\dfrac{3x^4y^5}{z^9}\right)^3$ $\boxed{44.}$ $\left(\dfrac{2x^5y^2}{4x^3y^7}\right)^4$ 45. $\left(\dfrac{9a^5b^4}{18ab^{11}}\right)^3$

46. $\dfrac{x^{-4}y^2}{x^5y^{-3}}$ 47. $\dfrac{a^2b^{-2}}{a^{-4}b^5}$ 48. $\left(\dfrac{6y^{-3}}{12y^{-5}}\right)^3$ 49. $\dfrac{x^{-4}y^2}{x^{-6}y^5}$

50. $\left(\dfrac{y^{-4}z^{-3}}{y^{-5}z^4}\right)^2$ $\boxed{51.}$ $\left(\dfrac{3x^{-2}}{12x^{-4}}\right)\left(\dfrac{16x^{-5}}{x}\right)$ 52. $\left(\dfrac{a^{-2}b^3c^0}{a^3b^{-4}c^2}\right)\left(\dfrac{a^{-4}b^{-1}c^2}{a^5b^2}\right)$ 53. $\left(\dfrac{3^2a^{-3}b^3}{6a^{-5}b^{-2}}\right)\left(\dfrac{12a^{-7}b^{-3}}{a^2b^5}\right)$

54. $\left(\dfrac{4a^{-3}}{12a^{-5}}\right)^{-3}$ 55. $\left(\dfrac{xy^0}{z^{-2}}\right)^{-4}$ 56. $\left(\dfrac{2a^3b^{-4}}{6a^5b^2}\right)^{-2}$ 57. $\left(\dfrac{4x^2y^3z^{-2}}{12x^5y^{-2}z^0}\right)^{-3}$

$\boxed{58.}$ $(3x^2y^{-2})^2(x^{-4}y^3)^3$ 59. $(2a^3b^{-4})^3(a^{-2}b^5)^3$ 60. $(x^{-2}y^4)^{-3}(x^{-1}y^{-2})^4$ 61. $(a^3b^{-2}c^{-4})^{-2}(a^{-2}b^3c^0)^3$

62. $(ab^{-4}c^2)^{-5}(a^{-2}b^0c^3)^{-3}$ 63. $(x^2y^{-4})^{-3}(xy^3z^4)^{-2}$

Perform the indicated operations and leave the answer as a product with no fractions and each variable occurring only once.

Examples $(a^{1-n})^{-3}$ $\dfrac{x^{2n-7}}{x^{n-6}}$

Solutions
$= a^{(1-n)(-3)}$
$= a^{-3+3n}$
$= a^{3n-3}$

$= x^{(2n-7)-(n-6)}$
$= x^{2n-7-n+6}$
$= x^{n-1}$

64. $x^{2n+3}x^{2-n}$ 65. $a^{b-4}a^{2b-1}$ 66. $(a^{1-2n})^{-2}$ 67. $(x^{4-3n})^{-2}$ 68. $\dfrac{x^{n-6}}{x^{n-3}}$

69. $\dfrac{a^{2n-1}}{a^{n-5}}$ 70. $\dfrac{x^ny^{2n+1}}{x^{n-3}y^{n-4}}$ 71. $\dfrac{a^{2n}b^{3n-2}}{a^{n-1}b^{n-5}}$ 72. $\left(\dfrac{x^n}{x^{2n-1}}\right)^{-2}$ 73. $\left(\dfrac{x^{2n}}{x^{n+1}}\right)^{-3}$

Express the following numbers in scientific notation. See example 3–3 F.

74. 65,000,000 75. 155,000 76. 0.00012 77. 0.0863 78. -0.0567

79. The mass of the moon is 8,060,000,000,000,000,000,000 tons.

80. General Motors reported a gross earnings of \$7,230,000,000.

81. A cu in. equals 0.0005787 cu ft.

82. 43,560 sq ft equals an acre.

83. A gram equals 0.0022046 pound.

84. 46,656 cu in. equals a cu yd.

85. A kilogram equals 0.001102 ton.

86. A particular gauge of copper wire has a resistance of 0.00006203 ohms/lb.

87. The distance light travels in an hour is 669,600,000 miles.

88. 3,785.3 milliliters equals a gal.

Convert the following numbers in scientific notation to their standard form. See example 3–3 G.

$\boxed{89.}$ -4.37×10^{-2} 90. 7.61×10^{-7} 91. 4.99×10^{6} 92. 7.23×10^{0} 93. 4.83×10^{4}

Multiply or divide the following using scientific notation. Leave the answer in scientific form, $a \times 10^n$, where a is rounded to two decimal places. See example 3–3 H.

Example $(473)(0.0000579)$

Solution

$= (4.73 \times 10^2)(5.79 \times 10^{-5})$	Scientific notation
$= (4.73)(5.79) \times (10^2 \cdot 10^{-5})$	Commutative and associative properties
$= 27.3867 \times 10^{-3}$	Multiplication
$= 2.73867 \times 10^1 \times 10^{-3}$	Scientific notation
$= 2.74 \times 10^{-2}$	Answer rounded to 2 decimal places

94. $5.23 \times 10^9 \cdot 1.073 \times 10^6$ 95. $5.12 \times 10^6 \cdot 6.2 \times 10^4$ 96. $8.473 \times 10^3 \cdot 8.4 \times 10^{-4}$

97. $1.673 \times 10^{-4} \cdot 7.5 \times 10^{-6}$ 98. $(6.23 \times 10^2) \div (5.73 \times 10^4)$ 99. $(1.47 \times 10^3) \div (8.03 \times 10^1)$

100. $(7.82 \times 10^{-2}) \div (5.6 \times 10^5)$ 101. $(7.23 \times 10^5) \div (6.075 \times 10^{-9})$ 102. $0.876 \cdot 21.46$

103. $0.000476 \cdot 0.0053$ 104. $0.000000089 \cdot 0.145$

Solve the following word problems, leave the answer in scientific notation rounded to two decimal places.

105. The amount of stress on a piece of metal is given by stress $= \dfrac{\text{force producing the stress}}{\text{area of the surface}}$. Find the stress on a piece of metal where the force is 452.6 kg and the area is 0.000763 sq cm.

106. If the volume of a right circular cylinder is given by $V = \pi r^2 h$, find the volume of a right circular cylinder that has radius $r = 0.0073$ m and height $h = 12$ m.

107. A unit of light intensity is a lumen. If 1 lumen equals 0.001496 watts, how many lumens are there in 4,760,000 watts?

Review exercises

Perform the indicated addition and subtraction. See section 1–6.

1. $5a + 7a$ 2. $7x^2 - 4x^2$ 3. $9ab - 3ab$

Perform the indicated multiplication. See section 3–2.

4. $a^2(2a + 3)$ 5. $3a^2b(2a^2 + 3ab - b^2)$ 6. $(a + 2b)(x - 3y)$

7. $(3x + 4y)(a - 2b)$ 8. $(a + 1)(b + 1)$

3–4 ■ Common factors and factoring by grouping

Prime numbers

In section 3–2, we studied the procedure for multiplying polynomials. Now we will study the reversal of that process, which is called **factoring.** Factoring polynomials is necessary to deal with algebraic fractions, since, as we have seen with arithmetic fractions, the denominators of the fractions must be in factored form to reduce fractions or to find the **least common denominator** (LCD). Factoring is also useful as a means of finding the solution of certain types of equations.

We saw in chapter 1 that whenever we multiply any two real numbers, a and b, a and b are called **factors** of the **product** ab. For example, 2 and 3 are factors of 6 because $2 \cdot 3 = 6$. The first type of factoring that we will consider is factoring a natural number into its prime factors.

The set of natural numbers greater than 1 can be divided into two types of numbers, prime numbers and composite numbers. First, a **prime number** *is any natural number greater than 1 that is divisible only by itself and by 1.* For example,

$$2, 3, 5, 7, 11, 13$$

are all prime numbers since each is a natural number greater than 1 and divisible only by 1 and the number itself. Second, a **composite number** *is any natural number greater than 1 that is not a prime number.* For example,

$$4, 9, 28, 42$$

are composite numbers since 4 is divisible by 2; 9 is divisible by 3; 28 is divisible by 2, 4, 7, and 14; 42 is divisible by 2, 3, 6, 7, 14, and 21.

When a composite number is stated as a product of only prime numbers, we call this the **prime factor form** or **completely factored form** of the composite number. We can also state negative integers in a completely factored form by first factoring out -1 and then factoring the resulting positive integer into its prime factors.

■ Example 3–4 A

Express the following numbers in prime factor form.

1. $28 = 4 \cdot 7 = 2 \cdot 2 \cdot 7 = 2^2 \cdot 7$

Note Whenever a prime number appears as a factor more than once, we write it in exponential form.

2. $42 = 6 \cdot 7 = 2 \cdot 3 \cdot 7$

3. $23 = 23$

Note The number 23 is a prime number, therefore it cannot be expressed as a product of only prime factors. It would be incorrect to write $23 \cdot 1$ since 1 is not a prime number.

4. $120 = 10 \cdot 12 = 2 \cdot 5 \cdot 2 \cdot 2 \cdot 3 = 2^3 \cdot 3 \cdot 5$ ■

The greatest common factor

The first type of factoring that we will do involves finding the **greatest common factor** (GCF) of the polynomial. Recall the statement of the distributive property.

$$a(b + c) = ab + ac$$

$a(b + c)$ is called the *factored form* of $ab + ac$. The greatest common factor consists of the following:

Greatest common factor

1. The greatest integer that is a common factor of all the numerical coefficients, and
2. The variable factor(s) common to every term, each raised to the least power to which they were raised in any of the terms.

Factoring →

Polynomial *(terms)*	**Distributive Property** *(determine the GCF)*	**Factored Form** *(factors)*
$5x + 15$	$\mathbf{5} \cdot x + \mathbf{5} \cdot 3$	$5(x + 3)$
$12ab - 6ac$	$\mathbf{6a} \cdot 2b - \mathbf{6a} \cdot c$	$6a(2b - c)$
$x^5 + 4x^3 - 2x^2$	$\mathbf{x^2}x^3 + \mathbf{x^2} \cdot 4x - \mathbf{x^2} \cdot 2$	$x^2(x^3 + 4x - 2)$

← Multiplying

We were able to determine the greatest common factor of these polynomials by inspection. In some problems, this may not be possible, and the following procedure will be necessary. Factor the polynomial $15x^5y^2 + 30x^3y^4$.

Factoring the greatest common factor

Step 1 Factor each term such that it is the product of primes and variables to powers.

$$15x^5y^2 \qquad\qquad 30x^3y^4$$
$$3 \cdot 5 \cdot x^5 \cdot y^2 \qquad\qquad 2 \cdot 3 \cdot 5 \cdot x^3 \cdot y^4$$

Step 2 Write down all factors that are common to *every* term.

$$3 \cdot 5 \cdot x \cdot y$$

Note We do not have 2 as part of our greatest common factor since it does not appear in *all* of the terms.

Step 3 Take the factors in step 2 and raise them to the *least* power to which they were raised in any of the terms.

$$3^1 \cdot 5^1 \cdot x^3 \cdot y^2 = 15x^3y^2$$

This is the greatest common factor (GCF).

Step 4 Find the multinomial factor (the polynomial within the parentheses) by dividing each term of the polynomial being factored by the GCF.

$$\frac{15x^5y^2}{15x^3y^2} = x^2 \quad \text{and} \quad \frac{30x^3y^4}{15x^3y^2} = 2y^2$$

$$(x^2 + 2y^2)$$

Step 5 We can now write the polynomial in its factored form.

$$\begin{aligned} &15x^5y^2 + 30x^3y^4 \\ &= 15x^3y^2 \cdot x^2 + 15x^3y^2 \cdot 2y^2 \\ &= 15x^3y^2(x^2 + 2y^2) \end{aligned}$$

Completely factored form

Notice that $15x^5y^2 + 30x^3y^4$ could also be factored into

$$15(x^5y^2 + 2x^3y^4) \text{ or } 30\left(\frac{1}{2}x^5y^2 + x^3y^4\right) \text{ or } x^3(15x^2y^2 + 30y^4)$$

This allows room for a given polynomial to be factored in many ways, unless some restrictions are placed on the procedure. We wish to factor each polynomial in a unique manner that will not permit such variations in the results.

Completely factored form

A polynomial with integer coefficients will be considered to be in **completely factored form** when it satisfies the following criteria:

1. The polynomial is written as a product of polynomials with integer coefficients.
2. None of the polynomial factors other than the monomial factor can be factored further.

We see that $15x^3y^2(x^2 + 2y^2)$ is the **completely factored form** of the expression $15x^5y^2 + 30x^3y^4$. All of the coefficients are integers and none of the factors other than the monomial, $15x^3y^2$, can still be written as the product of two polynomials with integer coefficients.

In general, whenever we factor a monomial out of a polynomial, we factor the monomial in such a way that it has a positive coefficient. We should realize that we can also factor out the opposite, or negative, of this common factor. In our previous example, we could have factored out $-15x^3y^2$ and the completely factored form would have been

$$-15x^3y^2(-x^2 - 2y^2)$$

Observe that the only change in our answer when we factor out the opposite of the common factor is that the signs of all the terms inside the parentheses change.

Example 3–4 B

Write in completely factored form.

1. $12x^5 + 9x^2$

$= 3x^2(\quad + \quad)$ — Multinomial factor will have as many terms as the original expression

GCF is $3x^2$

$= 3x^2(4x^3 + 3)$ — Completely factored form

$\frac{9x^2}{3x^2}$

$\frac{12x^5}{3x^2}$

If we wanted to check the answer, we would apply the distributive property and perform the multiplication as follows:

$$3x^2(4x^3 + 3) = 3x^2 \cdot 4x^3 + 3x^2 \cdot 3 \qquad \text{Distributive property}$$
$$= 12x^5 + 9x^2. \qquad \text{Carry out the multiplication}$$

2. $9r^3s^2 + 15r^2s^4 + 3rs^2$

$$\begin{aligned} &= 3^2 \cdot r^3 \cdot s^2 + 3 \cdot 5 \cdot r^2 \cdot s^4 + 3 \cdot r \cdot s^2 && \text{Factor each term} \\ &= 3rs^2 \cdot 3r^2 + 3rs^2 \cdot 5rs^2 + 3rs^2 \cdot 1 && \text{Determine the GCF} \\ &= 3rs^2(3r^2 + 5rs^2 + 1) && \text{Completely factored form} \end{aligned}$$

Note In example 2, the last term in the factored form is 1. This occurs when a term and the GCF are the same, that is, whenever we are able to factor all the numbers and variables out of a given term.

3. $a(x + 2y) + b(x + 2y)$

The quantity $(x + 2y)$ is common to both terms. We then factor the common quantity out of each term and place the remaining factors from each term in a second parentheses.

$$a(x + 2y) + b(x + 2y)$$
$$= \underbrace{(x + 2y)}_{\text{Common factor}}\underbrace{(a + b)}_{\text{Remaining factors}}$$

▶ ***Quick check*** Write $18y^4 + 12y^2$ in completely factored form.

Factoring by grouping

Consider the example $ax + ay + bx + by$. We observe that this is a four-term polynomial and we will *try* to factor it by grouping.

$$ax + ay + bx + by = (ax + ay) + (bx + by)$$

There is a common factor of a in the first two terms and a common factor of b in the last two terms.

$$(ax + ay) + (bx + by) = a(x + y) + b(x + y)$$

The quantity $(x + y)$ is a common factor to both terms. Factoring it out, we have

$$a(x + y) + b(x + y) = (x + y)(a + b)$$

Therefore we have factored the polynomial by grouping.

Factoring a four-term polynomial by grouping

1. Arrange the four terms so that the first two terms have a common factor and the last two terms have a common factor.
2. Determine the GCF of each pair of terms and factor it out.
3. If step 2 produces a common binomial factor in each term, factor it out.
4. If step 2 does not produce a common binomial factor in each term, try grouping the terms of the original polynomial in a different way.
5. If step 4 does not produce a common binomial factor in each term, the polynomial will not factor by this procedure.

■ Example 3–4 C

Factor completely.

1. $2ac - ad + 4bc - 2bd = (2ac - ad) + (4bc - 2bd)$ Group in pairs
$= a(2c - d) + 2b(2c - d)$ Factor out the GCF
$= (2c - d)(a + 2b)$ Factor out the common binomial

2. $6ax + by + 2ay + 3bx = 6ax + 2ay + 3bx + by$ Rearrange the terms
$= (6ax + 2ay) + (3bx + by)$ Group in pairs
$= 2a(3x + y) + b(3x + y)$ Factor out the GCF
$= (3x + y)(2a + b)$ Factor out the common binomial

Note Sometimes the terms must be rearranged such that the pairs will have a common factor, as we did in example 2.

▶ ***Quick check*** Factor $3a^3 - a^2 + 9a - 3$ completely. ■

Mastery points

Can you

- Determine the greatest common factor of a polynomial?
- Factor the greatest common factor from a polynomial?
- Factor a four-term polynomial by grouping?

Exercise 3–4

Express the following numbers in prime factor form. If a number is prime, so state. See example 3–4 A.

1. 12 **2.** 15 **3.** 24 **4.** 28 **5.** 56 **6.** 60 **7.** 41 **8.** 43 **9.** 39

Supply the missing factors or terms.

Examples $-3a + a^3b = -a(? - ?)$ $x^2y - z^3 = -(? + ?)$ $3a^2 - 9b = ?(-a^2 + 3b)$

Solutions $= -a(3 - a^2b)$ $= -(-x^2y + z^3)$ $= -3(-a^2 + 3b)$

10. $5x - 3 = -(? + ?)$
11. $2a - 3b = -(? + ?)$
12. $2x^2 - 6y = -2(? + ?)$
13. $3a^3 - 9b^2 = -3(? + ?)$
14. $6a - 8b - 12c = 2(? - ? - ?)$
15. $-4x^3 - 36xy + 16xy^2 = -4(? + ? - ?)$
16. $5x^2y + 15x^3y^2 + 25x^2y^2 = -5x^2y(? - ? - ?)$
17. $3a^3b^2 + 12a^2b^3 + 15a^4b^2 = -3a^2b^2(? - ? - ?)$
18. $-ab^2 - ac^2 = ?(b^2 + c^2)$
19. $3x^2 - 9xy = ?(-x + 3y)$
20. $-10a^2b^2 + 15ab - 20a^3b^3 = ?(-2ab + 3 - 4a^2b^2)$
21. $-24RS - 16R + 32R^2 = ?(3S + 2 - 4R)$
22. $3x^n - 2x^ny = ?(3 - 2y)$

Write in completely factored form. Assume that all variables used as exponents represent positive integers. See examples 3–4 B and C.

Examples $18y^4 + 12y^2$

Solutions $= 2 \cdot 3^2y^4 + 2^2 \cdot 3y^2$ Factor each term
$= 6y^2(\quad + \quad)$ Determine the GCF
$= 6y^2(3y^2 + 2)$ Completely factored form

$3a^3 - a^2 + 9a - 3$
$= (3a^3 - a^2) + (9a - 3)$ Group in pairs
$= a^2(3a - 1) + 3(3a - 1)$ Factor out the GCF
$= (3a - 1)(a^2 + 3)$ Factor out the common binomial

College Loan CLC Corporation

Need more money for college expenses?

The CLC Private Loan℠ can get you up to **$40,000** a year for college-related expenses.

Here's why the CLC Private Loan℠ is a smart choice:

- ☑ Approved borrowers are sent a check within four business days
- ☑ Get $1000 - $40,000 each year
- ☑ Interest rates as low as prime + 0% (8.66% APR)
- ☑ Quick and easy approval process
- ☑ No payments until after graduating or leaving school

CLICK HERE

or call **800.311.9615.**

We are available 24 hours a day, 7 days a week.

8.66% APR is based on a Prime Rate of 8.25% as of August 1, 2006.

23. $5x^2 + 10xy - 20y$
24. $8a - 12b + 16c$
25. $18ab - 27a + 3ac$
26. $15x^2 - 27y^2 + 12$
27. $15R^2 - 21S^2 + 36T$
28. $8x^3 + 4x^2$
29. $3R^2S - 6RS^2 + 12RS$
30. $16x^3y - 3x^2y^2 + 24x^2y^3$
31. $12x^4y^3 - 8x^4y^2 + 16x^3y$
32. $24a^3b^2 - 3a^2b^2 + 12a^2b^5$
33. $3x^2y - 6xy^4 + 15x^3y^2$
34. $a(x - 3y) + b(x - 3y)$
35. $3a(x - y) + b(x - y)$
36. $5a(3x - 1) + 10(3x - 1)$
37. $14b(a + c) - 7(a + c)$
38. $21x(1 + 2z) - 35y(1 + 2z)$
39. $4ab(2x + y) - 8ac(2x + y)$
40. $2x(y - 16) - (y - 16)$
41. $5a(b + 3) - (b + 3)$
42. $15a^3b^2(x + y) + 30a^2b^5(x + y)$
43. $x^ny^2 + x^nz$
44. $x^{3n} + x^{2n} + x^n$
45. $x^{2n}y^{2n} - x^ny^n$
46. $x^{n+4} + x^4$
47. $y^{n+3} - y^3$
48. $ac + ad - 2bc - 2bd$
49. $2ax + 6bx - ay - 3by$
50. $2ax + bx - 4ay - 2by$
51. $2ax + 3bx + 8ay + 12by$
52. $5ax - 3by + 15bx - ay$
53. $2ax^2 - 3b - bx^2 + 6a$
54. $20x^2 - 3yz + 5xz - 12xy$
55. $3ac - 2bd - 6ad + bc$
56. $6ac + 3bd - 2ad - 9bc$
57. $2x^3 + 15 + 10x^2 + 3x$
58. $3x^3 - 6x^2 + 5x - 10$
59. $8x^3 - 4x^2 + 6x - 3$

Solve the following word problems.

60. The area of the surface of a cylinder is determined by $A = 2\pi rh + 2\pi r^2$, where r is the radius and h is the height. Factor the right member.
61. The total surface area of a right circular cone is given by $A = \pi rs + \pi r^2$, where r is the base radius and s is the slant height. Factor the right member.
62. The equation for the height of a rocket fired vertically upward into the air is given by $S = 560t - 16t^2$, where the rocket is S feet from the ground after t seconds. Factor the right member.
63. If you have P dollars in a savings account where the yearly compounded rate is r percent, then the amount of money in the account at the end of one year can be written as $A = P + Pr$. Factor the right member.
64. In engineering, the equation for deflection of a beam is given by

$$Y = \frac{2wx^4}{48EI} - \frac{3\ell wx^3}{48EI} - \frac{\ell^3 wx}{48EI}$$

Factor the right member.

Review exercises

Perform the indicated multiplication. See section 3–2.

1. $(a + 3)(a + 4)$
2. $(x - 5)(x - 2)$
3. $(x + 4)(x - 6)$
4. $(x - 3)(x + 70)$
5. $(x - 4)(x + 4)$
6. $(a + 5)(a - 5)$
7. $(x + 3)^2$
8. $(x - 4)^2$

3–5 ■ Factoring trinomials of the form $x^2 + bx + c$ and perfect square trinomials

Determining when a trinomial will factor

In section 3–2, we learned how to multiply two binomials as follows:

$$\overset{\text{Factors}}{(a + 2)(a + 8)} = a^2 + 8a + 2a + 16 = \overset{\text{Terms}}{a^2 + 10a + 16}$$

$$\xrightarrow{\text{Multiplying}}$$

In this section, we are going to reverse the procedure and factor the trinomial.

Terms		Factors
$a^2 + 10a + 16$	$=$	$(a + 2)(a + 8)$

$$\xrightarrow{\text{Factoring}}$$

The following group of trinomials will enable us to see how a trinomial factors.

$$16 = 2 \cdot 8$$

1. $a^2 + 10a + 16 \quad = \quad (a + 2)(a + 8)$

$$10 = 2 + 8$$

$$16 = (-2) \cdot (-8)$$

2. $a^2 - 10a + 16 \quad = \quad (a - 2)(a - 8)$

$$-10 = (-2) + (-8)$$

$$-16 = (-2) \cdot 8$$

3. $a^2 + 6a - 16 \quad = \quad (a - 2)(a + 8)$

$$6 = (-2) + 8$$

$$-16 = 2 \cdot (-8)$$

4. $a^2 - 6a - 16 \quad = \quad (a + 2)(a - 8)$

$$-6 = 2 + (-8)$$

In general,

$$(x + m)(x + n) = x^2 + (m + n)\,x + m \cdot n$$

The trinomial $x^2 + bx + c$ will factor only if there are two integers, which we will call m and n, such that $m + n = b$ and $m \cdot n = c$.

$$\begin{array}{ccc} m + n & & m \cdot n \\ x^2 + bx & + & c = (x + m)(x + n) \end{array}$$

Factoring a trinomial of the form $x^2 + bx + c$

1. Factor out the GCF. If there is a common factor, make sure to include it as part of the final factorization.
2. Determine if the trinomial is factorable by finding m and n such that $m + n = b$ and $m \cdot n = c$. If m and n do not exist, we conclude that the trinomial will not factor.
3. Using the m and n values from step 2, write the trinomial in factored form.

The signs (+ or −) for m and n

1. If c is positive, then m and n both have the same sign as b.
2. If c is negative, then m and n have different signs and the one with the greater absolute value has the same sign as b.

Example 3–5 A

Factor completely each trinomial.

1. $x^2 + 14x + 24 \qquad m + n = 14 \quad \text{and} \quad m \cdot n = 24$
   Since $b = 14$ and $c = 24$ are both positive, then m and n are both positive.

List of the factorizations of 24	Sum of the factors of 24
$1 \cdot 24$	$1 + 24 = 25$
$2 \cdot 12$	$2 + 12 = 14$ ← Correct sum
$3 \cdot 8$	$3 + 8 = 11$
$4 \cdot 6$	$4 + 6 = 10$

   The m and n values are 2 and 12. The factorization is

   $x^2 + 14x + 24 = (x + 2)(x + 12)$

Note The commutative property allows us to write the factors in any order. That is $(x + 2)(x + 12) = (x + 12)(x + 2)$.

2. $a^2 - 3a - 18 \qquad m + n = -3 \quad \text{and} \quad m \cdot n = -18$
   Since $b = -3$ and $c = -18$ are both negative, then m and n have different signs and the one with the greater absolute value is negative.

Factorizations of −18, where the negative sign goes with the factor with the greater absolute value	Sum of the factors of −18
$1 \cdot (-18)$	$1 + (-18) = -17$
$2 \cdot (-9)$	$2 + (-9) = -7$
$3 \cdot (-6)$	$3 + (-6) = -3$ ← Correct sum

   The m and n values are 3 and -6. The factorization is

   $a^2 - 3a - 18 = (a + 3)(a - 6)$

3. $5a - 24 + a^2$
   It is easier to identify b and c if we write the trinomial in descending powers of the variable, which is called standard form.

   $a^2 + 5a - 24 \qquad m + n = 5 \quad \text{and} \quad m \cdot n = -24$

   Since $b = 5$ is positive and $c = -24$ is negative, m and n have different signs and the one with the greater absolute value is positive.

Factorizations of −24, where the positive factor is the one with the greater absolute value	Sum of the factors of −24
$(-1) \cdot 24$	$(-1) + 24 = 23$
$(-2) \cdot 12$	$(-2) + 12 = 10$
$(-3) \cdot 8$	$(-3) + 8 = 5$ ← Correct sum
$(-4) \cdot 6$	$(-4) + 6 = 2$

   The m and n values are -3 and 8. The factorization is

   $a^2 + 5a - 24 = (a - 3)(a + 8)$

4. $x^2 + 6x + 12 \qquad m + n = 6$ and $m \cdot n = 12$
Since $b = 5$ and $c = 12$ are both positive, m and n are both positive.

Factorizations of 12	Sum of the factors of 12
$1 \cdot 12$	$1 + 12 = 13$
$2 \cdot 6$	$2 + 6 = 8$
$3 \cdot 4$	$3 + 4 = 7$

No sum equals 6

Since none of the factorizations of 12 add to 6, there is no pair of integers (m and n) and the trinomial will not factor using integer coefficients. We call this a **prime polynomial.**

5. $2a^3 - 18a^2 + 40a = 2a(a^2 - 9a + 20) \qquad$ common factor of $2a$
To complete the factorization, we see if the trinomial $a^2 - 9a + 20$ will factor. We need to find m and n that add to -9 and multiply to 20. The values are -4 and -5. The completely factored form is

$$2a^3 - 18a^2 + 40a = 2a(a - 4)(a - 5)$$

Note A common error when the polynomial has a common factor is to factor it out but to forget to include it as one of the factors in the completely factored form.

6. $x^2y^2 - 8xy + 15$
Rewriting the polynomial as $(xy)^2 - 8xy + 15$, we want to find values for m and n that add to -8 and multiply to 15. The values are -3 and -5. The factorization is

$$x^2y^2 - 8xy + 15 = (xy - 3)(xy - 5)$$

7. $x^2 + xy - 6y^2 \qquad m + n = 1$ and $m \cdot n = -6 \quad m$ and n are -2 and 3.
We replace xy with $-2y$ and $3y$. The factorization is

$$x^2 + xy - 6y^2 = (x - 2y)(x + 3y)$$

8. $x^2(a - b) + 6x(a - b) + 8(a - b)$ — Common factor of $a - b$
$= (a - b)(x^2 + 6x + 8)$ — The trinomial $x^2 + 6x + 8$ has m and n values of 4 and 2
$= (a - b)(x + 4)(x + 2)$ — Completely factored form

9. $(x + y)^2 + 6(x + y) + 8$
If we let a represent $x + y$, then using the property of substitution, the factorization would be

$$\begin{array}{ccccccc} a^2 & +\ 6 & a & +\ 8 = & (a & +\ 2) & (a & +\ 4) \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ (x+y)^2 & +\ 6 & (x+y) & +\ 8 = & [(x+y) & +\ 2] & [(x+y) & +\ 4] \end{array}$$

Substitute $x + y$ for a

$$= (x + y + 2)(x + y + 4)$$

Remove inner parentheses

▶ ***Quick check*** Factor $c^2 - 9c + 14$ and $a^2(x - 2) + 7a(x - 2) + 12(x - 2)$ completely.

Perfect square trinomials

Two of the special products that we studied in section 3–2 were the squares of a binomial. We now restate those special products.

$$a^2 + 2ab + b^2 = (a + b)^2$$

and

$$a^2 - 2ab + b^2 = (a - b)^2$$

The right members of these two equations are called the **squares of binomials,** and the left members are called **perfect square trinomials.** Perfect square trinomials can always be factored by our m and n procedure from this section if the coefficient of the squared term is 1, or by the procedure in the next section if the coefficient is not 1. However if we observe that the first and last terms of a trinomial are perfect squares, we should see if the trinomial will factor as the square of a binomial. To factor a trinomial as a pefect square trinomial, three conditions need to be met.

Conditions for factoring a perfect square trinomial

1. The first term must have a positive coefficient and be a perfect square, a^2.
2. The last term must have a positive coefficient and be a perfect square, b^2.
3. The middle term must be twice the product of the bases of the first and last terms, $2ab$ or $-2ab$.

We observe that

$$\begin{aligned} &9x^2 + 12x + 4 \\ &= \underbrace{(3x)^2}_{\text{Condition 1}} + \underbrace{2(3x)(2)}_{\text{Condition 3}} + \underbrace{(2)^2}_{\text{Condition 2}}. \end{aligned}$$

Therefore it is a perfect square trinomial and factors into

$$(3x + 2)^2$$

Example 3–5 B

The following examples show the factoring of some other perfect square trinomials.

		Condition 1		*Condition 3*		*Condition 2*	*Square of a binomial*
1. $x^2 + 10x + 25$	$=$	$(x)^2$	$+$	$2(x)(5)$	$+$	$(5)^2$	$= (x + 5)^2$
2. $a^2 - 12a + 36$	$=$	$(a)^2$	$-$	$2(a)(6)$	$+$	$(6)^2$	$= (a - 6)^2$
3. $4b^2 + 20b + 25$	$=$	$(2b)^2$	$+$	$2(2b)(5)$	$+$	$(5)^2$	$= (2b + 5)^2$
4. $9x^2 - 24x + 16$	$=$	$(3x)^2$	$-$	$2(3x)(4)$	$+$	$(4)^2$	$= (3x - 4)^2$

▶ ***Quick check*** Factor $4x^2 + 20x + 25$ and $9x^2 - 6x + 1$ completely.

Mastery points

Can you

- Determine two integers whose product is one number and whose sum is another number?
- Recognize when the trinomial $x^2 + bx + c$ will factor and when it will not?
- Factor trinomials of the form $x^2 + bx + c$?
- Always remember to look for the greatest common factor before applying any of the factoring rules?
- Factor perfect square trinomials?

Exercise 3–5

Write in completely factored form. Assume all variables used as exponents represent positive integers. See example 3–5 A.

Example $c^2 - 9c + 14$

Solution $m + n = -9$ and $m \cdot n = 14$

Since $b = -9$ is negative and $c = 14$ is positive, m and n are both negative.

List the factorizations of 14	Sum of the factors of 14
$(-1)(-14)$	$(-1) + (-14) = -15$
$(-2)(-7)$	$(-2) + (-7) = -9$ ← Correct sum

The m and n values are -2 and -7. The factorization is

$c^2 - 9c + 14 = (c - 2)(c - 7)$

1. $x^2 + 13x - 30$
2. $a^2 - 14a + 24$
3. $y^2 + 5y - 24$
4. $a^2 + 8a + 12$
5. $x^2 - 2x - 24$
6. $y^2 - y - 12$
7. $3x^2 + 33x - 36$
8. $3y^2 - 18y - 48$
9. $4x^2 - 4x - 24$
10. $5y^2 - 15y - 55$
11. $3y^2 - 27y + 12$
12. $4s^2 - 28s + 12$
13. $x^2y^2 - 4xy - 21$
14. $x^2y^2 - 3xy - 18$
15. $x^2y^2 + 13xy + 12$
16. $a^2b^2 + 13ab - 30$
17. $x^2y^2 - 14xy + 24$
18. $y^2z^2 + 5yz - 24$
19. $4a^2b^2 + 24ab + 32$
20. $3x^2y^2 + 21xy + 30$
21. $-2x^2y^2 + 6xy + 20$
22. $-3x^2y^2 + 6xy + 45$
23. $a^2 + 7ab + 10b^2$
24. $x^2 - 12xy + 32y^2$
25. $x^2 + 11xy + 24y^2$
26. $y^2 - 4yz - 5z^2$
27. $a^2 + 2ab - 35b^2$
28. $y^2 - 4yz - 12z^2$
29. $x^2 + 10xy + 16y^2$
30. $x^{2n} + 5x^n + 6$
31. $x^{2n} + 9x^n + 14$
32. $x^{2n} - 4x^n - 12$
33. $x^{2n} - 8x^n + 15$

Write in completely factored form. See example 3–5 B.

Examples		**Solutions** Condition 1		Condition 3		Condition 2		Square of a binomial
$25x^2 + 20x + 4$	=	$(5x)^2$	+	$2(5x)(2)$	+	$(2)^2$	=	$(5x + 2)^2$
$9x^2 - 6x + 1$	=	$(3x)^2$	−	$2(3x)(1)$	+	$(1)^2$	=	$(3x - 1)^2$

34. $a^2 + 16a + 64$
35. $b^2 - 12b + 36$
36. $x^2 + 14x + 49$
37. $c^2 + 18c + 81$
38. $4a^2 + 20a + 25$
39. $9y^2 + 12y + 4$
40. $x^2 + 10x + 25$
41. $x^2 - 14xy + 49y^2$
42. $9a^2 - 12ab + 4b^2$
43. $4x^2 - 12xy + 9y^2$
44. $25a^2 + 10ab + b^2$
45. $9x^2 + 30xy + 25y^2$
46. $a^2 - 16ab + 64b^2$

Write in completely factored form. See example 3–5 A.

Example $a^2(x - 2) + 7a(x - 2) + 12(x - 2)$

Solution $= (x - 2)(a^2 + 7a + 12)$ Common factor of $x - 2$, m and n values are 3 and 4
$= (x - 2)(a + 3)(a + 4)$ Completely factored form

$\boxed{47.}$ $x^2(a - 2b) + 8x(a - 2b) + 12(a - 2b)$
48. $a^2(x - 3y) + 14a(x - 3y) + 24(x - 3y)$
49. $x^2(y + 2z) - 13x(y + 2z) + 30(y + 2z)$
50. $a^2(2b - c) + a(2b - c) - 12(2b - c)$
51. $x^2(3y - z) + 15x(3y - z) + 36(3y - z)$
52. $a^2(2b + 3c) - 6a(2b + 3c) - 27(2b + 3c)$
53. $a^2(3x - y) - 7a(3x - y) - 60(3x - y)$
54. $x^2(2y + z) + 2x(2y + z) - 63(2y + z)$
$\boxed{55.}$ $(a + b)^2 - 4(a + b) - 12$
56. $(2a - b)^2 + (2a - b) - 30$
57. $(y + 3z)^2 - 5(y + 3z) - 14$
58. $(2a + 3b)^2 + 8(2a + 3b) + 12$
59. $(R + 5S)^2 - 6(R + 5S) + 8$
60. $(x - 4y)^2 - 10(x - 4y) + 21$

Solve the following word problems.

61. If a rectangle has a perimeter of 52 inches and an area of 153 square inches, the dimensions of the rectangle can be found by factoring the expression $W^2 - 26W + 153$. Factor this expression.

62. If the length of a rectangle is 3 meters more than three times its width and its area is 168 square meters, then the dimensions of the rectangle can be found by factoring the expression $3W^2 + 3W - 168$. Factor this expression.

63. When two capacitors are connected in parallel, the total capacitance of the circuit, C_t, is given by $C_t = C_1 + C_2$. If the number of microfarads in C_1 is the square of the number in C_2 and the total capacitance is 42 microfarads, then the expression to find the capacitance C_2 is given by $C_2^2 + C_2 - 42$. Factor this expression.

Review exercises

Factor completely. See section 3–4.

1. $3x(2x - 1) + 5(2x - 1)$
2. $2a(3a + 1) + (3a + 1)$
3. $4x(2x - 3) - (2x - 3)$
4. $5x(x + 4) - 3(x + 4)$

Perform the indicated operations. See section 3–2.

5. $(2x + 1)(3x + 2)$
6. $(5a - 1)(2a + 3)$
7. $(b + 4)(3b - 7)$
8. $(3a + 2)(3a - 2)$

3–6 ■ Factoring trinomials of the form $ax^2 + bx + c$

How to factor trinomials

We are going to factor trinomials of the form $ax^2 + bx + c$. This is called the **standard form** of a trinomial, where we have a single variable and the terms of the polynomial are arranged in descending powers of that variable. Letters a, b, and c in our standard form represent integer constants. For example,

$$4x^2 + 20x + 21$$

is a trinomial in standard form, where $a = 4$, $b = 20$, and $c = 21$.

Consider the product

$$(2x + 3)(2x + 7)$$

By multiplying these two quantities together and combining like terms, we get a trinomial.

$$\begin{aligned}(2x + 3)(2x + 7) &= 4x^2 + 14x + 6x + 21\\ &= 4x^2 + 20x + 21\end{aligned}$$

To completely factor the trinomial $4x^2 + 20x + 21$ entails reversing the procedure to get

$$(2x + 3)(2x + 7)$$

The trinomial $ax^2 + bx + c$ will factor with integer coefficients if we can find a pair of integers (m and n) whose sum is equal to b, and whose product is equal to $a \cdot c$. In the trinomial $4x^2 + 20x + 21$, b is equal to 20, and $a \cdot c$ is $4 \cdot 21 = 84$. Therefore we want $m + n = 20$ and $m \cdot n = 84$. The values for m and n are 14 and 6.

If we observe the multiplication process in our example, we see that m and n appear as the coefficients of the middle terms that are to be combined for our final answer.

$$\begin{aligned}(2x + 3)(2x + 7) &= 4x^2 + 14x + 6x + 21\\ &= 4x^2 + 20x + 21\end{aligned}$$

This is precisely what we do with the m and n values. We replace the coefficient of the middle term in the trinomial with these values. In our example, m and n are 14 and 6, and we replace 20 with these numbers.

$$4x^2 + 20x + 21 = 4x^2 + \overbrace{14x + 6x}^{20x} + 21$$

The next step is to group the first two terms and the last two terms.

$$(4x^2 + 14x) + (6x + 21)$$

Now we factor out what is common in each pair. We see that the first two terms contain the common factor $2x$ and the last two terms contain the common factor 3.

$$2x(2x + 7) + 3(2x + 7)$$

When we reach this point, what is inside the parentheses in each term will be the same. Since the quantity $(2x + 7)$ represents just one number and this number is common to both terms, we can factor it out.

$$\underbrace{2x(2x + 7)}_{\text{Common to both terms}} + 3\underbrace{(2x + 7)}$$

Having factored out what is common, what is left in each term is placed in a second parentheses.

$$2x(2x + 7) + 3(2x + 7)$$
$$\underset{\text{Common factor}}{(2x + 7)}\underset{\text{Remaining factors}}{(2x + 3)}$$

The trinomial is factored.

A summary of the steps follows:

Factoring a trinomial

Step 1 Determine if the trinomial $ax^2 + bx + c$ is factorable by finding m and n such that $m \cdot n = a \cdot c$ and $m + n = b$. If m and n do not exist, we conclude that the trinomial will not factor.

Step 2 Replace the middle term, bx, by the sum of mx and nx.

Step 3 Place parentheses around the first and second terms and around the third and fourth terms. Factor out what is common to each pair.

Step 4 Factor out the common quantity of each term and place the remaining factors from each term in a second parentheses.

We determine the signs ($+$ or $-$) for m and n in a fashion similar to that of the previous section.

The signs ($+$ or $-$) for m and n

1. If $a \cdot c$ is positive, then m and n both have the same sign as b.
2. If $a \cdot c$ is negative, then m and n have different signs and the one with the greater absolute value has the same sign as b.

Example 3–6 A

Express the following trinomials in completely factored form. If the trinomial will not factor, so state.

1. $6x^2 + 13x + 6$

Step 1 $m \cdot n = 6 \cdot 6 = 36$ and $m + n = 13$

We determine by inspection that m and n are 9 and 4.

Step 2	$= 6x^2 + \overbrace{9x + 4x}^{13x} + 6$	Replace $13x$ with $9x$ and $4x$
Step 3	$= (6x^2 + 9x) + (4x + 6)$	Group the first two terms and the last two terms
	$= 3x(2x + 3) + 2(2x + 3)$	Factor out what is common to each pair
Step 4	$= (2x + 3)(3x + 2)$	Factor out the common quantity

Note The order in which we place m and n into the problem will not change the answer.

Alternate

Step 2	$= 6x^2 + \overbrace{4x + 9x}^{13x} + 6$	Replace $13x$ with $4x$ and $9x$
Step 3	$= (6x^2 + 4x) + (9x + 6)$	Group the first two terms and the last two terms
	$= 2x(3x + 2) + 3(3x + 2)$	Factor out what is common to each pair
Step 4	$= (3x + 2)(2x + 3)$	Factor out the common quantity

We see that the outcome in step 4 is the same regardless of the order of m and n in the problem.

2. $3x^2 + 14x + 8$

Step 1 $m \cdot n = 3 \cdot 8 = 24$ and $m + n = 14$
m and n are 2 and 12.

Step 2	$= 3x^2 + \overbrace{2x + 12x}^{14x} + 8$	Replace $14x$ with $2x$ and $12x$
Step 3	$= (3x^2 + 2x) + (12x + 8)$	Group the first two terms and the last two terms
	$= x(3x + 2) + 4(3x + 2)$	Factor out what is common to each pair
Step 4	$= (3x + 2)(x + 4)$	Factor out the common quantity

3. $6a^2 - 11a + 4$

Step 1 $m \cdot n = 6 \cdot 4 = 24$ and $m + n = -11$
m and n are -3 and -8.

Step 2	$= 6a^2 \overbrace{- 3a - 8a}^{-11a} + 4$	Replace $-11a$ with $-3a$ and $-8a$
Step 3	$= (6a^2 - 3a) + (-8a + 4)$	Group the first two terms and the last two terms
	$= 3a(2a - 1) - 4(2a - 1)$	Factor out what is common to each pair

In the last two terms, we have 4 or -4 as the greatest common factor. We factor out -4 so that we will have the same quantity inside the second parentheses.

Step 4 $= (2a - 1)(3a - 4)$ Factor out the common quantity

Note In the third step, if the third term is preceded by a minus sign, we will usually factor out the negative factor.

4. $4a^2 - 9a - 6$

$m \cdot n = 4 \cdot (-6) = -24$ and $m + n = -9$
The m and n values are not obvious by inspection.

Note If you cannot determine the m and n values by inspection, then you should use the following systematic procedure to list all the possible factorizations of $a \cdot c$. This way you will either find m and n or verify that the trinomial will not factor.

1. Take the natural numbers $1, 2, 3, 4, \cdots$ and divide them into the $a \cdot c$ product. Those that divide in evenly we write as a factorization using the correct m and n signs.

Factorization of -24, where the negative sign goes with the factor with the greater absolute value

$$\left.\begin{matrix} 1 \cdot (-24) \\ 2 \cdot (-12) \\ 3 \cdot (-8) \\ 4 \cdot (-6) \end{matrix}\right\} \qquad \left.\begin{matrix} (-6) \cdot 4 \\ (-8) \cdot 3 \\ (-12) \cdot 2 \\ (-24) \cdot 1 \end{matrix}\right\}$$

We note that the top four factorizations are the same as the bottom four. Therefore we need only perform this procedure until the factors repeat

$$\begin{matrix} 4 \cdot (-6) \\ (-6) \cdot 4 \end{matrix}$$

Factors repeat

2. Find the sum of the factorizations of $a \cdot c$. If there is a sum equal to b, the trinomial will factor. If there is no sum equal to b, then the trinomial will not factor with integer coefficients.

Factorizations of -24	Sum of the factors of -24	
$1 \cdot (-24)$	$1 + (-24) = -23$	
$2 \cdot (-12)$	$2 + (-12) = -10$	
$3 \cdot (-8)$	$3 + (-8) = -5$	Passed -9
$4 \cdot (-6)$	$4 + (-6) = -2$	
	No sum equals -9	

Since none of the factorizations of -24 also add to -9, there is no pair of integers (m and n) and the trinomial will not factor. Therefore, $4a^2 - 9a - 6$ is a **prime polynomial.**

Note Regardless of the signs of m and n, the column of values of the sum of the factors will either be increasing or decreasing. Therefore once the desired value has been passed, the process can be stopped and the trinomial will not factor.

▶ ***Quick check*** Express $3x^2 + 5x + 2$ and $4x^2 - 11x + 6$ in completely factored form, if possible.

Factoring by inspection—an alternative approach

In the first part of this section, we studied a systematic procedure for determining if a trinomial will factor and how to factor it. In many instances, we are able to determine how the trinomial will factor by inspecting the problem rather than applying this procedure.

Factoring by inspection is accomplished as follows: Factor $8a + 3a^2 + 5$.

Step 1 Write the trinomial in standard form.

$3a^2 + 8a + 5$ Arrange terms in descending powers of a

Step 2 Determine the possible combinations of first-degree factors of the first term.

$(3a \quad)(a \quad)$ The only factorization of $3a^2$ is $3a \cdot a$

Step 3 Combine with the factors of step 2 all the possible factors of the last term.

$(3a \quad 5)(a \quad 1)$
$(3a \quad 1)(a \quad 5)$ The only factorization of 5 is $5 \cdot 1$

Step 4 Determine the possible symbol (+ or −) between the terms in each binomial.

$(3a + 5)(a + 1)$
$(3a + 1)(a + 5)$

The rules of signed numbers given in chapter 1 provide the answer to step 4.

1. If the third term is preceded by a + sign and the middle term is preceded by a + sign, then the symbols will be

   $(\quad + \quad)(\quad + \quad).$

2. If the third term is preceded by a + sign and the middle term is preceded by a − sign, then the symbols will be

   $(\quad - \quad)(\quad - \quad).$

3. If the third term is preceded by a − sign, then the symbols will be

   $(\quad + \quad)(\quad - \quad)$
   or
   $(\quad - \quad)(\quad + \quad).$

Note It is assumed that the first term is preceded by a + sign or no sign. If it is preceded by a − sign, these rules could still be used if a (-1) is first factored out of all the terms.

Step 5 Determine which factors, if any, yield the correct middle term.

$(3a + 5)(a + 1)$

$+5a$ $\quad (+5a) + (+3a) = +8a$ Correct middle term

$+3a$

$(3a + 1)(a + 5)$

$+a$ $\quad (+a) + (15a) = +16a$

$+ 15a$

The first set of factors gives us the correct middle term. Therefore $(3a + 5)(a + 1)$ is the factorization of $3a^2 + 8a + 5$.

Example 3–6 B

Factor the following trinomials by inspection.

1. $x^2 + 7x + 12$

Step 1 $x^2 + 7x + 12$ Standard form

Step 2 $(x \quad)(x \quad)$ The only factorization of x^2 is $x \cdot x$

Step 3 $(x \quad 12)(x \quad 1)$ The factorizations of 12 are $1 \cdot 12$, $6 \cdot 2$, and $3 \cdot 4$
$(x \quad 6)(x \quad 2)$
$(x \quad 3)(x \quad 4)$

Step 4 $(x + 12)(x + 1)$ Using the rules of signed numbers, determine the possible signs between the terms
$(x + 6)(x + 2)$
$(x + 3)(x + 4)$

Step 5 $(x + 12)(x + 1)$

$+12x$ $\quad (+12x) + (+x) = +13x$

$+x$

$(x + 6)(x + 2)$

$+6x$ $\quad (+6x) + (+2x) = +8x$

$+2x$

$(x + 3)(x + 4)$

$+3x$ $\quad (+3x) + (+4x) = +7x$
Correct middle term

$+4x$

The factorization of $x^2 + 7x + 12$ is $(x + 3)(x + 4)$.

2. $6x^2 + 17x + 7$

Step 1 $6x^2 + 17x + 7$ Standard form

Step 2 $(6x \quad)(x \quad)$ $6x^2 = 6x \cdot x$ or $3x \cdot 2x$
$(3x \quad)(2x \quad)$

Step 3 $(6x \quad 7)(x \quad 1)$ The only factorization of 7 is $7 \cdot 1$
$(6x \quad 1)(x \quad 7)$
$(3x \quad 7)(2x \quad 1)$
$(3x \quad 1)(2x \quad 7)$

Step 4 $(6x + 7)(x + 1)$ Using the rules of signed numbers, determine the possible signs between the terms
$(6x + 1)(x + 7)$
$(3x + 7)(2x + 1)$
$(3x + 1)(2x + 7)$

Step 5 $(6x + 7)(x + 1)$

$+7x$ $\qquad (+7x) + (+6x) = +13x$

$+6x$

$(6x + 1)(x + 7)$

$+x$ $\qquad (+x) + (+42x) = +43x$

$+42x$

$(3x + 7)(2x + 1)$

$+14x$ $\qquad (+14x) + (+3x) = +17x$ Correct middle term

$+3x$

$(3x + 1)(2x + 7)$

$+2x$ $\qquad (+2x) + (+21x) = +23x$

$+21x$

Hence, $(3x + 7)(2x + 1)$ is the factorization of $6x^2 + 17x + 7$.

3. $19x - 7 + 6x^2$

Step 1 $6x^2 + 19x - 7$ Standard form

Step 2 $(6x \quad)(x \quad)$ $\quad 6x^2 = 6x \cdot x$ or $2x \cdot 3x$
$(2x \quad)(3x \quad)$

Step 3 $(6x \quad 7)(x \quad 1)$ $\quad$ The only factorization of 7 is $7 \cdot 1$
$(6x \quad 1)(x \quad 7)$
$(2x \quad 7)(3x \quad 1)$
$(2x \quad 1)(3x \quad 7)$

Step 4 $(6x + 7)(x - 1)$ or $(6x - 7)(x + 1)$ $\quad$ Using the rules of signed numbers, determine the possible signs between the terms
$(6x + 1)(x - 7)$ or $(6x - 1)(x + 7)$
$(2x + 7)(3x - 1)$ or $(2x - 7)(3x + 1)$
$(2x + 1)(3x - 7)$ or $(2x - 1)(3x + 7)$

Step 5 $(6x + 7)(x - 1)$ or $(6x - 7)(x + 1)$

$+7x \qquad\qquad -7x$

$-6x \qquad\qquad +6x$

$(+7x) + (-6x) = +x$ or $(-7x) + (+6x) = -x$

$(6x + 1)(x - 7)$ or $(6x - 1)(x + 7)$

$+x \qquad\qquad -x$

$-42x \qquad\qquad +42x$

$(+x) + (-42x) = -41x$ or $(-x) + (+42x) = +41x$

$$(2x + 7)(3x - 1) \text{ or } (2x - 7)(3x + 1)$$

Inner products: $+21x$ and $-21x$; outer products: $-2x$ and $+2x$.

$$(+21x) + (-2x) = +19x \text{ or } (-21x) + (+2x) = -19x$$

Correct middle term

$$(2x + 1)(3x - 7) \text{ or } (2x - 1)(3x + 7)$$

Inner products: $+3x$ and $-3x$; outer products: $-14x$ and $+14x$.

$$(+3x) + (-14x) = -11x \text{ or } (-3x) + (+14x) = +11x$$

The factorization of $6x^2 + 19x - 7$ is $(2x + 7)(3x - 1)$.

Mastery points

Can you

- Determine two integers whose product is one number and whose sum is another number?
- Recognize when a trinomial will factor and when it will not?
- Factor trinomials of the form $ax^2 + bx + c$?
- Always remember to look for the greatest common factor before applying any of the factoring rules?

Exercise 3–6

Factor completely each trinomial. If a trinomial will not factor, state that as the answer. See examples 3–6 A and B.

Example $3x^2 + 5x + 2$

Solution *Step 1* $m \cdot n = 3 \cdot 2 = 6$ and $m + n = 5$
m and n are 2 and 3.

Step 2 $= 3x^2 + \overbrace{2x + 3x}^{5x} + 2$ — Replace $5x$ with $2x$ and $3x$
Step 3 $= (3x^2 + 2x) + (3x + 2)$ — Group the first two terms and the last two terms
$= x(3x + 2) + 1(3x + 2)$ — Factor out what is common to each pair
Step 4 $= (3x + 2)(x + 1)$ — Factor out the common quantity

Example $4x^2 - 11x + 6$

Solution *Step 1* $m \cdot n = 4 \cdot 6 = 24$ and $m + n = -11$
m and n are -3 and -8.

Step 2 $= 4x^2 \overbrace{- 3x - 8x}^{-11x} + 6$ — Replace $-11x$ with $-3x$ and $-8x$
Step 3 $= (4x^2 - 3x) + (-8x + 6)$ — Group the first two terms and the last two terms
$= x(4x - 3) - 2(4x - 3)$ — Factor out what is common to each pair
Step 4 $= (4x - 3)(x - 2)$ — Factor out the common quantity

1. $3a^2 + 7a - 6$
2. $2y^2 + y - 6$
3. $4x^2 - 5x + 1$
4. $2z^2 + 3z + 1$
5. $x^2 + 18x + 32$
6. $2a^2 - 7a + 6$
7. $2y^2 - y - 1$
8. $5x^2 - 7x - 6$
9. $8x^2 - 17x + 2$
10. $9x^2 - 6x + 1$
11. $2a^2 - 11a + 12$
12. $5y^2 + 4y + 6$
13. $2x^2 + 13x + 18$
14. $6x^2 + 13x + 6$
15. $7x^2 + 20x - 3$
16. $4a^2 + 20a + 21$
17. $4x^2 - 4x - 3$
18. $4z^2 - 2z + 5$
19. $6x^2 - 23x - 4$
20. $9x^2 - 21x - 8$
21. $10z^2 + 9z + 2$
22. $10y^2 + 7y - 6$
23. $7x^2 - 3x + 6$
24. $2x^2 - 9x + 10$
25. $5x^2 - 9x - 2$
$\boxed{26.}$ $6x^2 + 21x + 18$
27. $6x^2 - 17x + 12$
28. $4y^2 + 10y + 6$
29. $3x^2 + 12x + 12$
30. $6x^2 + 7x - 3$
31. $3x^2 + 8x - 4$
32. $2x^2 + 6x - 20$
33. $6x^2 - 38x + 40$
34. $18x^2 + 15x - 18$
$\boxed{35.}$ $-4x^2 - 12x - 9$
36. $-9z^2 + 30z - 25$
37. $-7x^2 + 36x - 5$
38. $-3x^2 - 2x - 4$
39. $12x^2 + 13x - 4$
40. $15x^2 + 2x - 1$
$\boxed{41.}$ $4x^3 + 10x^2 + 4x$
42. $2x^3 - 6x^2 - 20x$
43. $18x^2 - 42x - 16$
44. $2x^3 + 15x^2 + 7x$
45. $8x^2 - 18x + 9$
46. $8y^2 - 14y - 15$

Review exercises

Perform the indicated multiplication. See section 3–2.

1. $(3a - b)(3a + b)$
2. $(x - 2y)(x + 2y)$
3. $(3x - 4y)(3x + 4y)$
4. $(x^2 + 5)(x^2 - 5)$
5. $(x^2 + 4)(x - 2)(x + 2)$
6. $(a + 2b)(a^2 - 2ab + 4b^2)$
7. $(x - 3y)(x^2 + 3xy + 9y^2)$
8. $(3a - 2b)(9a^2 + 6ab + 4b^2)$

3–7 ■ Other methods of factoring

The difference of two squares

We saw in section 3–2 that the product $(a + b)(a - b)$ is $a^2 - b^2$. We refer to the indicated product $(a + b)(a - b)$ as the *product* of the *sum* and *difference* of the same two terms. Notice that in one factor we *add* the terms and in the other we find the *difference* between these same terms. The product will *always* be the *difference of the squares of the two terms.*

To factor the **difference of two squares,** we reverse the equation from section 3–2.

The difference of two squares factors as

$$a^2 - b^2 = (a + b)(a - b)$$

Concept

Verbally we have

(1st term)^2 − (2nd term)^2 factors into
(1st term + 2nd term) · (1st term − 2nd term)

To use this factoring procedure, we must be able to recognize **perfect squares.** For example,

$$16, \quad x^2, \quad 25a^2, \quad 9y^4$$

are all perfect squares since

$$16 = 4 \cdot 4 = (4)^2$$
$$x^2 = x \cdot x = (x)^2$$
$$25a^2 = 5a \cdot 5a = (5a)^2$$
$$9y^4 = 3y^2 \cdot 3y^2 = (3y^2)^2$$

We see from the examples that *a perfect square is any quantity that can be written as an exact square of a rational quantity.*

This is the procedure we use for factoring the difference of two squares.

Factoring the difference of two squares

Step 1 Identify that we have a perfect square minus another perfect square.

Step 2 Rewrite the problem as a first term squared minus a second term squared.

$$(\text{1st term})^2 - (\text{2nd term})^2$$

Step 3 Factor the problem into the first term plus the second term times the first term minus the second term.

$$(\text{1st term} + \text{2nd term})(\text{1st term} - \text{2nd term})$$

Example 3–7 A

Write in completely factored form.

1. $x^2 - 16$ — Identify as the difference of squares
   $= (x)^2 - (4)^2$ — Rewrite as squares
   $= (x + 4)(x - 4)$ — Factor the binomial

2. $25a^2 - 4b^2$ — Identify as the difference of squares
   $= (5a)^2 - (2b)^2$ — Rewrite as squares
   $= (5a + 2b)(5a - 2b)$ — Factor the binomial

3. $2x^2 - 18y^2$ — Always look for the greatest common factor first
   $= 2(x^2 - 9y^2)$ — Common factor of 2
   $= 2[(x)^2 - (3y)^2]$ — Rewrite as squares
   $= 2(x + 3y)(x - 3y)$ — Factor the binomial

4. $x^4 - 81$ — Identify as the difference of squares
   $= (x^2)^2 - (9)^2$ — Rewrite as squares
   $= (x^2 + 9)(x^2 - 9)$ — Factor the binomial and inspect the factors
   $= (x^2 + 9)[(x)^2 - (3)^2]$ — Identify the second factor as the difference of squares
   $= (x^2 + 9)(x + 3)(x - 3)$ — Factor the second binomial

   Our first factorization of $x^4 - 81$ yielded the factors $x^2 + 9$ and $x^2 - 9$. On further inspection, we observe that $x^2 - 9$ is the difference of two squares.

Note In example 4, $x^2 + 9$ is called the sum of two squares and will not factor using real numbers.

5. $x^{2n} - 4$ — Identify as the difference of squares
   $= (x^n)^2 - (2)^2$ — Rewrite as squares
   $= (x^n + 2)(x^n - 2)$ — Factor the binomial

▶ ***Quick check*** Write $3b^2 - 12c^2$ in completely factored form.

The difference of two cubes

To factor problems involving the difference of two squares, we identified the two terms as squares and applied the procedure. Now we will factor the **difference of two cubes** in a similar fashion.

Consider the indicated product $(a - b)(a^2 + ab + b^2)$. If we carry out the multiplication, we have

$$\begin{aligned}(a - b)(a^2 + ab + b^2) &= a^3 + a^2b + ab^2 - a^2b - ab^2 - b^3\\ &= a^3 - b^3\end{aligned}$$

Therefore, $(a - b)(a^2 + ab + b^2) = a^3 - b^3$ and $(a - b)(a^2 + ab + b^2)$ is the factored form of $a^3 - b^3$.

The difference of two cubes factors as

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Concept

If we are able to write a two-term polynomial as a first term cubed minus a second term cubed, then it will factor as the difference of two cubes.

$$(\text{1st term})^3 - (\text{2nd term})^3 =$$
$$(\text{1st term} - \text{2nd term})[(\text{1st term})^2 + (\text{1st term} \cdot \text{2nd term}) + (\text{2nd term})^2]$$

This is the procedure we use for factoring the difference of two cubes.

Factoring the difference of two cubes

Step 1 Identify that we have a perfect cube minus another perfect cube.

Step 2 Rewrite the problem as a first term cubed minus a second term cubed.

$$(\text{1st term})^3 - (\text{2nd term})^3$$

Step 3 Factor the problem into the first term minus the second term times the first term squared plus the first term times the second term plus the second term squared.

$$(\text{1st term} - \text{2nd term})[(\text{1st term})^2 + (\text{1st term} \cdot \text{2nd term}) + (\text{2nd term})^2]$$

Example 3–7 B

Factor completely.

1. $a^3 - 8$

We rewrite both the terms as perfect cubes.

$a^3 - 8 = (a)^3 - (2)^3$

The first term is a and the second term is 2. Then we write down the procedure for factoring the difference of two cubes.

$$(\underset{\substack{\uparrow\\ \text{1st}}}{\quad} - \underset{\substack{\uparrow\\ \text{2nd}}}{\quad})[(\underset{\substack{\uparrow\\ \text{1st}}}{\quad})^2 + (\underset{\substack{\uparrow\\ \text{1st}}}{\quad})(\underset{\substack{\uparrow\\ \text{2nd}}}{\quad}) + (\underset{\substack{\uparrow\\ \text{2nd}}}{\quad})^2]$$

Now substitute a where the first term is and 2 where the second term is.

$$(\underset{\text{1st}}{a} - \underset{\text{2nd}}{2})[(\underset{\text{1st}}{a})^2 + (\underset{\text{1st}}{a})(\underset{\text{2nd}}{2}) + (\underset{\text{2nd}}{2})^2]$$

Finally, we simplify the second factor.

$(a - 2)(a^2 + 2a + 4)$

Therefore $a^3 - 8 = (a - 2)(a^2 + 2a + 4)$.

2. $40x^3 - 135y^3$ — Always look for the greatest common factor first

$= 5(8x^3 - 27y^3)$ — 5 is a common factor, identify the binomial

$= 5[(2x)^3 - (3y)^3]$ — Rewrite as cubes

$5(\quad - \quad)[(\quad)^2 + (\quad)(\quad) + (\quad)^2]$ — Factoring procedure ready for substitution

$5(2x - 3y)[(2x)^2 + (2x)(3y) + (3y)^2]$ — The first term is $2x$, the second term is $3y$

$= 5(2x - 3y)(4x^2 + 6xy + 9y^2)$ — Simplify within the second group

3. $a^{21} - 64b^{15}$ — Identify the difference of two cubes

$= (a^7)^3 - (4b^5)^3$ — Rewrite as cubes

$(\quad - \quad)[(\quad)^2 + (\quad)(\quad) + (\quad)^2]$ — Factoring procedure ready for substitution

$(a^7 - 4b^5)[(a^7)^2 + (a^7)(4b^5) + (4b^5)^2]$ — The first term is a^7, the second term is $4b^5$

$= (a^7 - 4b^5)(a^{14} + 4a^7b^5 + 16b^{10})$ — Simplify within the second group

Note In example 3, we observe that a variable raised to a power that is a multiple of 3 can be written as a cube by dividing the exponent by 3. The quotient is the exponent of the number inside the parentheses and the 3 is the exponent outside the parentheses. For example,

$$y^{12} = (y^4)^3 \text{ and } z^{24} = (z^8)^3$$

The sum of two cubes

If we carry out the indicated multiplication in $(a + b)(a^2 - ab + b^2)$, we have

$$\begin{aligned}(a + b)(a^2 - ab + b^2) &= a^3 - a^2b + ab^2 + a^2b - ab^2 + b^3 \\ &= a^3 + b^3\end{aligned}$$

Therefore $(a + b)(a^2 - ab + b^2) = a^3 + b^3$ and $(a + b)(a^2 - ab + b^2)$ is the factored form of $a^3 + b^3$.

The sum of two cubes factors as

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

Concept

If we are able to write a two-term polynomial as a first term cubed plus a second term cubed, then it will factor as the sum of two cubes.

$$(\text{1st term})^3 + (\text{2nd term})^3 =$$
$$(\text{1st term} + \text{2nd term})[(\text{1st term})^2 - (\text{1st term})(\text{2nd term}) + (\text{2nd term})^2]$$

This is the procedure we use for factoring the sum of two cubes.

Factoring the sum of two cubes

Step 1 Identify that we have a perfect cube plus another perfect cube.

Step 2 Rewrite the problem as a first term cubed plus a second term cubed.

$$(\text{1st term})^3 + (\text{2nd term})^3$$

Step 3 Factor the problem into the first term plus the second term times the first term squared minus the first term times the second term plus the second term squared.

$$(\text{1st term} + \text{2nd term})[(\text{1st term})^2 - (\text{1st term})(\text{2nd term}) + (\text{2nd term})^2]$$

Example 3–7 C

Factor completely.

1. $x^3 + 27$ — Identify the sum of two cubes

$= (x)^3 + (3)^3$ — Rewrite as cubes

$(\underset{\text{1st}}{\uparrow}\ +\ \underset{\text{2nd}}{\uparrow})[(\underset{\text{1st}}{\uparrow})^2 - (\underset{\text{1st}}{\uparrow})(\underset{\text{2nd}}{\uparrow}) + (\underset{\text{2nd}}{\uparrow})^2]$ — Factoring procedure ready for substitution

$(\underset{\text{1st}}{x} + \underset{\text{2nd}}{3})[(\underset{\text{1st}}{x})^2 - (\underset{\text{1st}}{x})(\underset{\text{2nd}}{3}) + (\underset{\text{2nd}}{3})^2]$ — The first term is x, the second term is 3

$= (x + 3)(x^2 - 3x + 9)$ — Simplify within the second group

2. $8x^3 + y^9$ — Identify the sum of two cubes

$= (2x)^3 + (y^3)^3$ — Rewrite as cubes

$(\ \ +\ \ )[(\ \ )^2 - (\ \ )(\ \ ) + (\ \ )^2]$ — Factoring procedure ready for substitution

$(2x + y^3)[(2x)^2 - (2x)(y^3) + (y^3)^2]$ — The first term is $2x$, the second term is y^3

$= (2x + y^3)(4x^2 - 2xy^3 + y^6)$ — Simplify within the second group

3. $a^3b^3 + 27c^3$ — Identify the sum of two cubes

$= (ab)^3 + (3c)^3$ — Rewrite as cubes

$(\ \ +\ \ )[(\ \ )^2 - (\ \ )(\ \ ) + (\ \ )^2]$ — Factoring procedure ready for substitution

$(ab + 3c)[(ab)^2 - (ab)(3c) + (3c)^2]$ — The first term is ab, the second term is $3c$

$= (ab + 3c)(a^2b^2 - 3abc + 9c^2)$ — Simplify within the second group

▶ ***Quick check*** Factor $27b^3 + c^{12}$ completely.

Mastery points

Can you

- Factor the difference of two squares?
- Factor the sum and the difference of two cubes?

Exercise 3–7

Write in completely factored form. Assume all variables used as exponents represent positive integers. See examples 3–7 A, B, and C.

Example $3b^2 - 12c^2$

Solution
$3(b^2 - 4c^2)$ — Common factor of 3 and identify the binomial
$3[(b)^2 - (2c)^2]$ — Rewrite as squares
$3(b + 2c)(b - 2c)$ — Factor the binomial

Example $27b^3 + c^{12}$

Solution
$(3b)^3 + (c^4)^3$ — Identify and rewrite as cubes
$(\quad + \quad)[(\quad)^2 - (\quad)(\quad) + (\quad)^2]$ — Factoring procedure ready for substitution
$(3b + c^4)[(3b)^2 - (3b)(c^4) + (c^4)^2]$ — The first term is $3b$, the second term is c^4
$(3b + c^4)(9b^2 - 3bc^4 + c^8)$ — Simplify within the second group

1. $a^2 - 49$
2. $36 - R^2$
3. $64 - S^2$
4. $4a^2 - 25b^2$
5. $36x^2 - y^4$
6. $x^4 - 4$
7. $16a^2 - 49b^2$
8. $144x^2 - 121y^2$
9. $8a^2 - 32b^2$
10. $3x^2 - 27y^2$
11. $50 - 2x^2$
12. $128a^2 - 2b^2$
13. $98a^2b^2 - 50x^2y^2$
14. $x^4 - y^4$
15. $a^4 - 81$
16. $16R^4 - 1$
17. $x^4 - 16y^4$
18. $x^{2n} - 1$
19. $a^{2n} - 4$
20. $x^{4n} - 25$
21. $x^{2n} - y^{2n}$
22. $x^{4n} - 16$
23. $x^{4n} - 81$
24. $a^2(x + 2y) - b^2(x + 2y)$
25. $4x^2(a + 5b) - y^2(a + 5b)$
26. $2a^2(x + y) - 8b^2(x + y)$
27. $3a^2(3x - y) - 27(3x - y)$
28. $4a(x^2 - y^2) - 8b(x^2 - y^2)$
29. $9x(4a^2 - b^2) - 3y(4a^2 - b^2)$
30. $(a + b)^2 - (x - y)^2$
31. $(2a + b)^2 - (x - 2y)^2$
32. $(2x - y)^2 - (x + y)^2$
33. $(3a - b)^2 - (2a - b)^2$
34. $(4x + 3y)^2 - (4x - 2y)^2$
35. $(x + 2y)^2 - (x - 3y)^2$
36. $R^3 - 8$
37. $27a^3 - b^3$
38. $8y^3 - 27$
39. $x^3 + y^3$
40. $27 + x^3$
41. $a^3 + 8b^3$
42. $64a^3 + b^3$
43. $8y^3 - 27x^3$
44. $64a^3 - 8$
45. $81a^3 - 3b^3$
46. $2a^3 + 16$
47. $3x^3 + 24$
48. $2a^3 - 16$
49. $64z^3 + 125$
50. $a^5 + 27a^2b^3$
51. $2a^3 - 54b^3$
52. $8x^2y^3 - x^5$
53. $R^5 + 64R^2S^3$
54. $54y^3 + 2z^3$
55. $3x^5 - 81x^2y^3$
56. $a^3b^9 - c^{15}$
57. $x^3y^{12} + z^9$
58. $x^{15}y^6 - 8z^9$
59. $a^{18}b^9 - 27c^3$
60. $2a^3b^3 + 16$
61. $3x^3y^6 + 81z^3$
62. $(a + 2b)^3 - c^3$
63. $(x + 3y)^3 + z^3$
64. $(2a - b)^3 + 8c^3$
65. $(4a + b)^3 - 27c^3$
66. $(2a + b)^3 - (x - y)^3$
67. $(a - 2b)^3 - (2x + y)^3$
68. $(3a + b)^3 - (a - 2b)^3$
69. $(3x - y)^3 + (2x + 3y)^3$
70. $(a - 2b)^3 - (a + b)^3$
71. $(2x - y)^3 + (x + y)^3$
72. $(5x - 2y)^3 + (3x + 2y)^3$
73. In engineering, the equation of transverse shearing stress in a rectangular beam is given in two forms:
    (a) $T = \dfrac{V}{8I}(h^2 - 4V_1^2)$ and
    (b) $T = \dfrac{3V}{2A}\left(1 - \dfrac{4V_1^2}{H^2}\right)$
    Factor the right member of each equation.

Review exercises

Write in completely factored form. See sections 3–4, 3–5, and 3–6.

1. $a^2 - 7a + 10$ **2.** $5ax + 2bx - 10ay - 4by$ **3.** $x^2 + 4xy + 4y^2$

4. $6a^2 - ab - b^2$ **5.** $5a^3 - 40a^2 + 75a$ **6.** $6x^2 + x - 12$

3–8 ■ Factoring: A general strategy

In this section, we will review the different methods of factoring that we have studied in the previous sections. The following outline gives a general strategy for factoring polynomials.

I. Factor out the greatest common factor.
Examples
1. $5x^3 - 25x^2 = 5x^2(x - 5)$
2. $x(a - 2b) + 2y(a - 2b) = (a - 2b)(x + 2y)$

II. Count the number of terms.
A. Two terms: Check to see if the polynomial is the difference of two squares, the difference of two cubes, or the sum of two cubes.
Examples
1. $x^2 - 25y^2 = (x + 5y)(x - 5y)$ Difference of two squares
2. $8a^3 - b^3 = (2a - b)(4a^2 + 2ab + b^2)$ Difference of two cubes
3. $m^3 + 64n^3 = (m + 4n)(m^2 - 4mn + 16n^2)$ Sum of two cubes

B. Three terms: Check to see if the polynomial is a perfect square trinomial. If it is not, use one of the general methods for factoring a trinomial.
Examples
1. $m^2 + 6m + 9 = (m + 3)^2$ Perfect square trinomial
2. $m^2 - 5m - 14 = (m - 7)(m + 2)$ General trinomial, leading coefficient of 1
3. $6x^2 + 7x - 20 = (2x + 5)(3x - 4)$ General trinomial, leading coefficient other than 1

C. Four terms: Check to see if we can factor by grouping.
Examples
1. $ax + 3a - 2bx - 6b = (a - 2b)(x + 3)$
2. $x^3 + 2x^2 - 3x - 6 = (x^2 - 3)(x + 2)$

III. Check to see if any of the factors we have written can be factored further. Any common factors that were missed in part I can still be factored out here.
Examples
1. $x^4 - 11x^2 + 28 = (x^2 - 4)(x^2 - 7)$
$= (x - 2)(x + 2)(x^2 - 7)$ Difference of two squares
2. $4a^2 - 36b^2 = (2a + 6b)(2a - 6b)$
$= 2(a + 3b)2(a - 3b) = 4(a + 3b)(a - 3b)$ Overlooked common factor

The following examples illustrate our strategy for factoring polynomials.

Example 3–8 A

Completely factor the following polynomials.

1. $12a^3 - 3ab^2$
   I. First, we look for the greatest common factor.

   $3a(4a^2 - b^2)$ — Common factor of $3a$

   II. The factor $4a^2 - b^2$ has two terms and is the difference of two squares.

   $3a(4a^2 - b^2) = 3a(2a + b)(2a - b)$ — Factoring the binomial

   III. After checking to see if any of the factors will factor further, we conclude that $3a(2a + b)(2a - b)$ is the completely factored form. Therefore

   $12a^3 - 3ab^2 = 3a(2a + b)(2a - b)$

2. $3x^2 + 7xy - 6y^2$
   I. There is no common factor (other than 1 or -1).
   II. Since this trinomial is not a perfect square, we use our general method for factoring trinomials.
   $m \cdot n = -18$ and $m + n = 7$, m and n are -2 and 9.

   $$\begin{aligned} &3x^2 + 7xy - 6y^2 \\ &\quad = 3x^2 - 2xy + 9xy - 6y^2 && \text{Replace } 7xy \text{ with } -2xy \text{ and } 9xy \\ &\quad = x(3x - 2y) + 3y(3x - 2y) && \text{Group the first two terms and the last two terms and factor out what is common to each pair} \\ &\quad = (3x - 2y)(x + 3y) && \text{Factor out the common quantity} \end{aligned}$$

   III. None of the factors will factor further.

   $3x^2 + 7xy - 6y^2 = (3x - 2y)(x + 3y)$

3. $2am^2 + bm^2 - 2an^2 - bn^2$
   I. There is no common factor (other than 1 or -1).
   II. The polynomial has four terms and we factor by grouping.

   $$\begin{aligned} &= (2am^2 + bm^2) + (-2an^2 - bn^2) && \text{Group in pairs} \\ &= m^2(2a + b) - n^2(2a + b) && \text{Factor out what is common to each pair} \\ &= (2a + b)(m^2 - n^2) && \text{Factor out the common quantity} \end{aligned}$$

   III. The factor $m^2 - n^2$ is the difference of two squares.

   $2am^2 + bm^2 - 2an^2 - bn^2 = (2a + b)(m + n)(m - n)$ — Completely factored form

4. $5x^3 + 5x^2 + 20x$
   I. The greatest common factor is $5x$.

   $5x(x^2 + x + 4)$ — Common factor of $5x$

   II. The trinomial $x^2 + x + 4$ will not factor.
   III. None of the factors will factor further.

   $5x^3 + 5x^2 + 20x = 5x(x^2 + x + 4)$ — Completely factored form

5. $a^8 + 8a^2b^3$
   I. The greatest common factor is a^2.

   $a^2(a^6 + 8b^3)$ — Common factor of a^2

II. The factor $a^6 + 8b^3$ is the sum of two cubes.

$a^2(a^6 + 8b^3) = a^2(a^2 + 2b)(a^4 - 2a^2b + 4b^2)$ Factoring the sum of cubes

III. None of the factors will factor further.

$a^8 + 8a^2b^3 = a^2(a^2 + 2b)(a^4 - 2a^2b + 4b^2)$ Completely factored form

6. $a^2(4 - x^2) + 8a(4 - x^2) + 16(4 - x^2)$

I. The greatest common factor is $4 - x^2$.

$(4 - x^2)(a^2 + 8a + 16)$ Common factor of $4 - x^2$

II. The factor $4 - x^2$ is the difference of two squares. The factor $a^2 + 8a + 16$ is a perfect square trinomial.

$(2 + x)(2 - x)(a + 4)^2$ Factoring the binomial and the trinomial

III. None of the factors will factor further.

$a^2(4 - x^2) + 8a(4 - x^2) + 16(4 - x^2)$
$= (2 + x)(2 - x)(a + 4)^2$ Completely factored form

▶ ***Quick check*** Factor $4x^2 - 36y^2$ completely.

Mastery points

Can you

- Factor out the greatest common factor?
- Factor the difference of two squares?
- Factor the difference of two cubes?
- Factor the sum of two cubes?
- Factor trinomials?
- Factor a four-term polynomial?
- Use the general strategy for factoring polynomials?

Exercise 3–8

Completely factor the following polynomials. If a polynomial will not factor, so state. See the outline of the general strategy for factoring polynomials and example 3–8 A.

Example $4x^2 - 36y^2$

Solution
I. $4x^2 - 36y^2 = 4(x^2 - 9y^2)$ Common factor of 4
II. $= 4(x + 3y)(x - 3y)$ Factoring the binomial
III. $4x^2 - 36y^2 = 4(x + 3y)(x - 3y)$ Completely factored form

1. $m^2 - 49$
2. $81 - x^2$
3. $x^2 + 6x + 5$
4. $a^2 + 11a + 10$
5. $7a^2 + 36a + 5$
6. $3x^2 + 13x + 4$
7. $2a^2 + 15a + 18$
8. $5b^2 + 16b + 12$
9. $a^2b^2 + 2ab - 8$
10. $x^2y^2 - 5xy - 14$
11. $27a^3 + b^3$
12. $x^3 + 64y^3$
13. $25x^2(3x + y) + 5a(3x + y)$
14. $36x^2(2a - b) - 12x(2a - b)$
15. $10x^2 - 20xy + 10y^2$
16. $6a^2 - 24ab - 48b^2$
17. $4m^2 - 16n^2$
18. $9x^2 - 36y^2$

19. $(a - b)^2 - (2x + y)^2$
20. $(3a + b)^2 - (x - y)^2$
21. $3x^6 - 81y^3$
22. $32a^4 - 4ab^9$
23. $12x^3y^2 - 18x^2y^2 + 16xy^4$
24. $9x^5y - 6x^3y^3 + 3x^2y^2$
25. $4x^2 - 9y^2$
26. $36 - a^2b^2$
27. $3ax + 6ay - bx - 2by$
28. $6am + 4bm - 3an - 2bn$
29. $27a^9 - b^3c^3$
30. $x^{12} - y^3z^6$
31. $5a^2 - 32a - 21$
32. $7a^2 + 16a - 15$
33. $a^4 - 5a^2 + 4$
34. $x^4 - 37x^2 + 36$
35. $4a^2 - 4ab - 15b^2$
36. $6x^2 + 7xy - 3y^2$
37. $y^4 - 16$
38. $z^4 - 81$
39. $4a^2 + 10a + 4$
40. $6x^2 + 18x - 60$
41. $(x + y)^2 - 8(x + y) - 9$
42. $(a - 2b)^2 + 7(a - 2b) + 10$
43. $6a^2 + 7a - 5$
44. $4x^2 + 17x - 15$
45. $4ab(x + 3y) - 8a^2b^2(x + 3y)$
46. $3x^2y(m - 4n) + 15xy^2(m - 4n)$
47. $4a^2 - 20ab + 25b^2$
48. $9m^2 - 30mn + 25n^2$
49. $80x^5 - 5x$
50. $3b^5 - 48b$
51. $3a^5b - 18a^3b^3 + 27ab^5$
52. $3x^3y^3 + 6x^2y^4 + 3xy^5$
53. $9a^2 - (x + 5y)^2$
54. $7x(a^2 - 4b^2) + 14(a^2 - 4b^2)$
55. $3x^2 + 8x - 91$
56. $3x^2 - 32x - 91$
57. $3x^5y^9 + 81x^2z^6$
58. $a^8b^4 + 27a^2b^7$
59. $a^2(9 - x^2) - 6a(9 - x^2) + 9(9 - x^2)$
60. $x^2(16 - b^2) + 10x(16 - b^2) + 25(16 - b^2)$

Review exercises

Find the solution set of the following equations. See section 2–1.

1. $x - 7 = 0$
2. $2x + 3 = 0$
3. $5x - 4 = 0$
4. $3x = 0$
5. $6 - 5x = 0$
6. $4 - 2x = 0$

Chapter 3 lead-in problem

The amount A of a radioactive substance remaining after time t can be found using the formula $A = A_0(0.5)^{t/n}$, where A_0 represents the original amount of radioactive material and n is the half-life given in the same units of time as t. If the half-life of radioactive carbon 14 is 5,770 years, how much radioactive carbon 14 will remain after 11,540 years if we start with 100 grams?

Solution

$A = A_0(0.5)^{t/n}$	Original formula
$A = (\quad)(0.5)^{(\quad)/(\quad)}$	Formula ready for substitution
$A = (100)(0.5)^{11{,}540/5770}$	Substitute
$A = (100)(0.5)^2$	Simplify the exponent
$A = (100)(0.25)$	Powers
$A = 25$	Multiply

There will be 25 grams of radioactive carbon 14 remaining after 11,540 years.

Chapter 3 summary

1. In the expression a^n, a is called the **base** and n the **exponent.**
2. The following are properties and definitions involving exponents.
   a. $a^n = \overbrace{a \cdot a \cdot a \cdots a}^{n \text{ factors of } a}$
   b. $a^m \cdot a^n = a^{m+n}$
   c. $(a^m)^n = a^{mn}$
   d. $(ab)^n = a^n b^n$
   e. $a^m \div a^n = \dfrac{a^m}{a^n} = a^{m-n}, a \neq 0$
   f. $a^{-n} = \dfrac{1}{a^n}, a \neq 0$
   g. $a^0 = 1, a \neq 0$
   h. $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}, b \neq 0$
3. When multiplying two multinomials, we multiply each term of the first multinomial by each term of the second multinomial and then combine like terms.
4. Three **special products** are
   $(a + b)^2 = a^2 + 2ab + b^2$
   $(a - b)^2 = a^2 - 2ab + b^2$
   $(a + b)(a - b) = a^2 - b^2$
5. The **scientific notation** form of a number Y is $Y = a \times 10^n$, where a is a number greater than or equal to 1 and less than 10, and n is an integer.
6. **A prime number** is any natural number greater than 1 that is divisible only by itself and by 1.
7. A polynomial with integer coefficients will be considered to be in **completely factored form** when it is expressed as the product of polynomials with integer coefficients and none of the factors except a monomial can still be written as the product of two polynomials with integer coefficients.
8. We try to factor **four-term polynomials** by grouping.
9. The **trinomial** $x^2 + bx + c$ will factor only if we can find a pair of integers, m and n, whose product is c and whose sum is b.
10. The **trinomial** $ax^2 + bx + c$ will factor only if we can find a pair of integers, m and n, whose product is $a \cdot c$ and whose sum is b.
11. Perfect square trinomials factor as
    $a^2 + 2ab + b^2 = (a + b)^2$
    and
    $a^2 - 2ab + b^2 = (a - b)^2$
12. The **difference of two squares** factors as
    $$a^2 - b^2 = (a + b)(a - b)$$
13. The **sum and difference of two cubes** factors as
    $$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$
    and
    $$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Chapter 3 error analysis

1. Exponential notation
   Example: $(-2)^4 = -(2 \cdot 2 \cdot 2 \cdot 2) = -16$
   Correct answer: 16
   What error was made? (*see page 16*)
2. Product of like bases
   Example: $a \cdot a^3 \cdot a^5 = a^{3+5} = a^8$
   Correct answer: a^9
   What error was made? (*see page 99*)
3. Power to a power
   Example: $(x^3)^5 = x^{3+5} = x^8$
   Correct answer: x^{15}
   What error was made? (*see page 99*)
4. Power to a power
   Example: $(4ab^3)^2 = 4a^2(b^3)^2 = 4a^2b^6$
   Correct answer: $16a^2b^6$
   What error was made? (*see page 100*)
5. Squaring a binomial
   Example: $(x + 9)^2 = (x)^2 + (9)^2 = x^2 + 81$
   Correct answer: $x^2 + 18x + 81$
   What error was made? (*see page 106*)
6. Factoring the sum of two cubes
   Example: $x^3 + 8 = (x + 2)^3$
   Correct answer: $(x + 2)(x^2 - 2x + 4)$
   What error was made? (*see page 144*)
7. Negative exponents
   Example: $5^{-2} = -(5)^2 = -25$
   Correct answer: $\dfrac{1}{25}$
   What error was made? (*see page 113*)
8. Negative exponents
   Example: $-6^{-3} = 6^3 = 216$
   Correct answer: $-\dfrac{1}{216}$
   What error was made? (*see page 113*)
9. Zero exponents
   Example: $5x^0 = 1$
   Correct answer: 5
   What error was made? (*see page 114*)
10. Completely factoring an expression
    Example: $6x^2 - 18x - 24 = (x - 4)(6x + 6)$
    Correct answer: $6(x - 4)(x + 1)$
    What error was made? (*see page 147*)

Chapter 3 critical thinking

Given the problem 48^2, determine a method by which you can square the 48 mentally.

Chapter 3 review

[3–1]

Perform the indicated operations.

1. $(-5x^2)(3x^3)$
2. $(2a^2b)(3ab^4)$
3. $(2a^3b^4c)^3$
4. $(xy^3z)^2(x^2y)^3$
5. $(3a^2)^2a^3 + (2a)^3a^4$
6. $(2b^5)^2 - (3b^2)^3b^2$

[3–2]

Perform the indicated multiplication and simplify.

7. $5a^2b(2a^2 - 3ab + 4b^2)$
8. $(2a - b)(a + b)$
9. $(y - 7)(y + 7)$
10. $(2a + b)^2$
11. $(a + b)(a^2 - 3ab + 2b^2)$
12. $(3x - 2)(x + 4) - (x - 3)^2$

[3–3]

Perform all indicated operations and leave the answers with only positive exponents. Assume that all variables represent nonzero real numbers.

13. $\dfrac{2a^2b^4}{2^3a^5b}$
14. $(3x^{-4})(2x^{-5})$
15. $(3x^{-2}y^3z^0)^2$
16. $\left(\dfrac{16x^4y^3}{4xy^8}\right)^3$
17. $\dfrac{a^5b^{-2}}{a^{-4}b}$
18. $\dfrac{2^{-3}a^0b^{-3}c^2}{4^{-1}a^{-3}b^{-2}c^{-5}}$
19. $(2x^3y^{-4})^2(x^{-2}y)^3$

[3–4]

Write in completely factored form.

20. $12x^3 - 18x^2$
21. $12a^4b - 4a^2b^3 + 24a^3b^2$
22. $x(a + b) - 2y(a + b)$
23. $10x^2(x - 3z) + 5x(x - 3z)$
24. $6ax - 3ay - 2bx + by$
25. $4ax + 6by + 8ay + 3bx$
26. $4ax + 2ay + 6bx + 3by$
27. $ax + 3bx - 2ay - 6by$
28. $6ax + by - 2bx - 3ay$
29. $2ax + 3by + 6bx + ay$

[3–5]

Write in completely factored form. If a polynomial will not factor, so state.

30. $a^2 + 14a + 24$
31. $b^2 - 9b + 14$
32. $2a^3 - 8a^2 - 10a$
33. $x^3 - x^2 - 6x$
34. $x^2y^2 + 10xy + 24$
35. $a^2b^2 - 8ab - 20$
36. $(x + 3y)^2 + (x + 3y) - 2$
37. $(x + 2y)^2 - 14(x + 2y) + 49$
38. $a^2 - ab - 2b^2$
39. $x^2 + 5xy + 6y^2$

[3–6]

Write in completely factored form. If a polynomial will not factor, so state.

40. $2a^2 - 7a + 6$
41. $8x^2 - 14x + 5$
42. $6y^2 - 5y - 4$
43. $3a^2 + 2a - 1$
44. $4x^2 + 11x - 3$
45. $4x^2 + 4x + 1$
46. $24a^2 + 22a + 3$
47. $8x^2 - 18x + 9$
48. $2x^2 + 15x + 18$
49. $6x^2 + 51x + 24$
50. $-2x^2 + 11x + 6$

[3–7]

Write in completely factored form.

51. $x^2 - 81$
52. $4a^2 - 36b^2$
53. $3a^2 - 27b^2$
54. $y^4 - 81$
55. $x^2(a + 2b) - y^2(a + 2b)$
56. $4a^2(x - 3z) - 9b^2(x - 3z)$

57. $a^3 + 8b^3$
58. $27x^3 - y^3$
59. $64a^3 + 8b^3$
60. $27x^5 + x^2y^3$
61. $2x^3 - 16b^3$
62. $x^{12}y^{15} - z^9$

[3–8]

Write in completely factored form.

63. $9x^2 + 24x + 16$
64. $a^6 - b^9$
65. $3a^3 - 75a$
66. $a^2(9 - x^2) + 12a(9 - x^2) + 36(9 - x^2)$
67. $6a^2 + 17a - 3$
68. $4mx - 8my + 3nx - 6ny$
69. $40x^4 + 5xy^3$
70. $a^3b^2 - 4a^2b^3 + 4ab^4$
71. $x^2 + 2xy - 8y^2$
72. $15a^2 + 4a - 4$

Chapter 3 cumulative test

Determine if the following statements are true or false.

[1–1] 1. $|-10| < 0$
[1–2] 2. $-3^2 = (-3)^2$
[1–1] 3. $J \subseteq Q$

Perform the indicated operations, if possible, and simplify.

[1–2] 4. $\dfrac{(-8)}{(-4)}$
[1–2] 5. $\dfrac{(-7)}{0}$
[1–2] 6. $(-2) - (-6)$
[1–2] 7. $(-2)(4)(0)(-6)$
[1–2] 8. $(-3)^4$
[1–4] 9. $48 - 24 \div 6 - 3 - 2^2$
[1–4] 10. $2[-3(10 - 7) - 12 + 4]$

Solve the following equations and inequalities.

[2–5] 11. $3x - 4 \geq x + 10$
[2–1] 12. $2(3x - 4) + 7 = 8x - 11$
[2–4] 13. $|3x + 4| = 5$
[2–6] 14. $|8 - 3x| < 9$
[2–6] 15. $|6x + 5| - 4 > 2$
[2–5] 16. $-4 < 3 - 2x < 6$
[2–5] 17. $-3 \leq 2x + 7 \leq 4$
[2–3] 18. Three times a number is subtracted from 43 and this result is less than 16. Find all numbers that satisfy this condition.
[2–3] 19. Earl has invested \$8,000 at a 9% rate. How much more must he invest at 12% to make the total income for one year from both investments a 10% rate?

Perform the indicated operations and simplify. Assume that all variables represent nonzero real numbers.

[3–1] 20. $(2a^2b)(3a^3b^4)$
[3–1] 21. $(2x^3y^4)^4$
[3–3] 22. $\dfrac{27a^5b^9c}{18a^2b^4c^7}$
[3–3] 23. $\dfrac{x^{-4}y^3}{x^2y^{-1}}$
[3–3] 24. $\left(\dfrac{3x^2y^4}{9x^5y}\right)^3$
[1–6] 25. $(5x^2 - 3x + 4) - (x^2 - x + 7)$
[3–3] 26. $(3a^{-3}b^2c^{-1})^{-3}$
[1–6] 27. $4ab - \{2a + 3b - [a - (2b - 3ab)]\}$
[3–2] 28. $(3a - b)^2$
[3–1] 29. $(2a^2b)^3(a^3b^2)^4$
[3–2] 30. $6x^4y^3(2x^2 - 3x^2y + 4y^2)$

Write in completely factored form. If a polynomial will not factor, so state.

[3–7] 31. $x^2 - 36$
[3–7] 32. $x^3 + 27y^3$
[3–6] 33. $2a^2 - 15a + 18$
[3–6] 34. $y^2 + 7y + 6$
[3–5] 35. $x^2y^2 - 2xy - 8$
[3–6] 36. $7x^2 - 34x - 5$
[3–4] 37. $2ax - 6ay + 3bx - 9by$
[3–4] 38. $15x^2(2a - b) + 5x(2a - b)$
[3–5] 39. $4x^2 - 20xy + 25y^2$
[3–7] 40. $3a^2 - 12b^2$

Solutions to trial exercise problems

24. $|1 - 2x| \le 5$

$$-5 \le 1 - 2x \le 5$$
$$-6 \le -2x \le 4$$
$$\frac{-6}{-2} \ge \frac{-2x}{-2} \ge \frac{4}{-2}$$
$$3 \ge x \ge -2$$
$$\{x|-2 \le x \le 3\}, [-2,3]$$

28. $|4x - 9| < 0$

$\emptyset$. The absolute value cannot be less than zero.

58. Let $x =$ the number, then $|3x - 4| \ge 11$.

$$3x - 4 \ge 11 \quad \text{or} \quad 3x - 4 \le -11$$
$$3x \ge 15 \qquad\qquad 3x \le -7$$
$$x \ge 5 \qquad\qquad x \le -\frac{7}{3}$$
$$\left\{x|x \le -\frac{7}{3} \text{ or } x \ge 5\right\}, \left(-\infty,-\frac{7}{3}\right] \cup [5,+\infty)$$

Review exercises

1. -16 **2.** 16 **3.** -16 **4.** 16 **5.** x^5 **6.** x^3 **7.** x^2
8. xy

Chapter 2 review

1. $\{8\}$ **2.** $\{6\}$ **3.** $\{28\}$ **4.** $\{4\}$ **5.** $\left\{\frac{15}{7}\right\}$ **6.** $\left\{\frac{10}{3}\right\}$ **7.** $\left\{\frac{5}{3}\right\}$
8. $\left\{-\frac{13}{5}\right\}$ **9.** $\{9\}$ **10.** $w = \frac{v}{\ell h}$ **11.** $t = \frac{v - k}{g}$
12. $d = \frac{D - R}{q}$ **13.** $b = \frac{ar^2 - v}{r^2}$ **14.** $v = \frac{2s + gt^2}{2t}$
15. $n = \frac{\ell - a + d}{d}$ **16.** $x = \frac{3y}{2}$ **17.** 12 **18.** 36
19. $\frac{33}{5},\frac{99}{5},\frac{3}{5}$ **20.** 11 feet, 30 feet **21.** \$15,000 at 10%; \$9,000 at 8% **22.** 40 cl of 42% solution, 60 cl of 12% solution
23. $\{-15,15\}$ **24.** $\left\{-\frac{17}{3},\frac{7}{3}\right\}$ **25.** $\left\{-\frac{3}{2},\frac{17}{2}\right\}$ **26.** $\{0,3\}$
27. $\left\{-7,-\frac{1}{5}\right\}$ **28.** $\left\{-\frac{11}{2},-\frac{1}{6}\right\}$ **29.** $\{x|x \le 6\}, (-\infty,6]$
30. $\{x|x > 16\}, (16,+\infty)$ **31.** $\left\{x|x > -\frac{9}{2}\right\}, \left(-\frac{9}{2},+\infty\right)$
32. $\left\{x|x \le \frac{9}{8}\right\}, \left(-\infty,\frac{9}{8}\right]$ **33.** $\left\{x|x > -\frac{3}{2}\right\}, \left(-\frac{3}{2},+\infty\right)$
34. $\left\{x|x \le \frac{25}{3}\right\}, \left(-\infty,\frac{25}{3}\right]$ **35.** $\left\{x|-\frac{7}{2} < x < 1\right\}, \left(-\frac{7}{2},1\right)$
36. $\left\{x|-\frac{4}{5} \le x \le 0\right\}, \left[-\frac{4}{5},0\right]$ **37.** $\{x|-5 < x < -2\}$, $(-5,-2)$ **38.** $\left\{x\left|\frac{2}{3} < x \le \frac{10}{3}\right.\right\}, \left(\frac{2}{3},\frac{10}{3}\right]$
39. $\{x|-1 \le x < 0\}, [-1,0)$ **40.** let $x =$ the number; $4x - 5 \ge 19$, $\{x|x \ge 6\}$, $[6,+\infty)$ **41.** let $x =$ the number; $22 < 3x + 7 < 34$, $\{x|5 < x < 9\}$, $(5,9)$
42. $\{x|x \le -10 \text{ or } x \ge 10\}, (-\infty,-10] \cup [10,+\infty)$
43. $\left\{x|-\frac{11}{2} < x < \frac{1}{2}\right\}, \left(-\frac{11}{2},\frac{1}{2}\right)$
44. $\left\{x|-\frac{6}{5} \le x \le \frac{8}{5}\right\}, \left[-\frac{6}{5},\frac{8}{5}\right]$
45. $\left\{x|x > \frac{1}{2} \text{ or } x < -4\right\}, (-\infty,-4) \cup \left(\frac{1}{2},+\infty\right)$
46. $\left\{x|x \le -\frac{4}{3} \text{ or } x \ge 2\right\}, \left(-\infty,-\frac{4}{3}\right] \cup [2,+\infty)$
47. $\left\{x|-\frac{9}{4} < x < \frac{15}{4}\right\}, \left(-\frac{9}{4},\frac{15}{4}\right)$
48. all real numbers, $\{x|x \in R\}, (-\infty,+\infty)$
49. $\left\{x|-\frac{3}{2} < x < \frac{5}{2}\right\}, \left(-\frac{3}{2},\frac{5}{2}\right)$
50. $\left\{x|x \le -\frac{9}{2} \text{ or } x \ge 2\right\}, \left(-\infty,-\frac{9}{2}\right] \cup [2,+\infty)$ **51.** $\emptyset$

Chapter 2 cumulative test

1. false **2.** false **3.** true **4.** true **5.** true **6.** true
7. true **8.** 24 **9.** -1 **10.** 12 **11.** 0 **12.** 37 **13.** -6
14. -49 **15.** commutative property of multiplication
16. commutative property of multiplication **17.** $\{1,2,3,4,5\}$
18. $\{4,6,8,9,10,11\}$ **19.** $\{10,12\}$ **20.** $\left\{\frac{7}{4}\right\}$
21. $P = -\frac{M}{\ell - x}$ or $\frac{M}{x - \ell}$
22. $P_2 = \frac{P + nP_1 + c}{n}$ **23.** $\{x|x \ge -5\}$
24. $\left\{x|x \le -\frac{11}{4} \text{ or } x \ge -\frac{1}{4}\right\}$ **25.** $\left\{\frac{21}{17}\right\}$ **26.** $\left\{x|x \ge \frac{15}{4}\right\}$
27. $\{-1,1\}$ **28.** $\left\{\frac{16}{7}\right\}$ **29.** $\emptyset$ **30.** $\left\{x|-1 \le x \le \frac{11}{3}\right\}$
31. 16,8,48 **32.** \$26,000 at 11%; \$14,000 at 8%

Chapter 3

Exercise 3–1

Answers to odd-numbered problems

1. $(-2)^4$, -2 base, 4 exponent **3.** x^5, x base, 5 exponent
5. $(2x)^4$, $2x$ base, 4 exponent **7.** $(x^2 + 3y)^3$, $x^2 + 3y$ base, 3 exponent **9.** -2^2, 2 base, 2 exponent **11.** a^9 **13.** y^3
15. $(-2)^6 = 64$ **17.** $(-2)^4 = 16$ **19.** -36 **21.** x^8
23. x^7y^5 **25.** $6a^3b^2$ **27.** $6x^5$ **29.** $24x^3y^9$ **31.** $2^6 = 64$
33. -729 **35.** 64 **37.** a^{12} **39.** $x^{10}y^5z^{15}$ **41.** $49s^8t^4$
43. $x^{36}y^{48}z^{32}$ **45.** a^9b^{17} **47.** $x^{13}y^{24}$ **49.** $-x^{10}y^{18}$
51. $-75x^{10}y^{11}$ **53.** $17x^9$ **55.** $-76a^{13}$ **57.** $9x^{10} - 8x^8$
59. $24a^{13} + 18a^{11}$ **61.** $3x^{17} + 96x^{14}$ **63.** a^{9b} **65.** a^{9b}
67. a^{5b+3} **69.** x^{3y+5} **71.** x^{15y^2} **73.** \$5,634.13
75. 2.5 grams

Solutions to trial exercise problems

18. $(-2)(-2^2) = (-2)(-4) = 8$ **32.** $(-2^2)^3 = (-4)^3 = -64$
44. $(3x^2y)^2(2xy^3) = 9x^4y^2 \cdot 2xy^3 = 18x^5y^5$
52. $(2a^2)^2a^3 + (3a)^3a^4 = 4a^4a^3 + 27a^3a^4 = 4a^7 + 27a^7 = 31a^7$
57. $(3x^5)^2 - (2x^2)^3x^2 = 9x^{10} - 8x^6x^2 = 9x^{10} - 8x^8$. The subtraction cannot be performed because we do not have like terms.
62. $x^{5n} \cdot x^{4n} = x^{5n+4n} = x^{9n}$
66. $x^{2n+1} \cdot x^{n+4} = x^{(2n+1)+(n+4)} = x^{2n+1+n+4} = x^{3n+5}$
70. $(a^{3n})^{4n} = a^{3n \cdot 4n} = a^{12n^2}$

Review exercises

1. -24 **2.** -9 **3.** 0 **4.** 25 **5.** $9ab$ **6.** $a^2 - 2a - 15$
7. $x^2 - 9$ **8.** $x^2 + 2y^2$

Exercise 3–2

Answers to odd-numbered problems

1. $2a^3 - 3a^2 + 4a$ **3.** $-6y^3 + 10y^2 - 8y$ **5.** $12a^4 - 6a^3b + 9a^2b^2$ **7.** $30x^3y^2 - 24x^2y^4 + 12x^2y^2$ **9.** $-10x^5y^5 - 35x^4y^{10} + 15x^5y^9$ **11.** $a^2 + 8a + 15$ **13.** $b^2 - b - 20$ **15.** $2x^2 + xy - y^2$ **17.** $10x^2 + 3xy - y^2$ **19.** $42x^2 - 2xy - 20y^2$
21. $x^2 + 6x + 9$ **23.** $9x^2 + 6xy + y^2$ **25.** $16x^2 + 24xy + 9y^2$
27. $4a^2 - 20a + 25$ **29.** $16x^2 - 24xy + 9y^2$ **31.** $x^2 - 9y^2$
33. $25a^2 - 4b^2c^2$ **35.** $x^3 - 2x^2y - xy^2 + 2y^3$
37. $15a^3 - 31a^2b + 23ab^2 - 6b^3$ **39.** $x^4 + x^3 - 5x^2 - 17x - 12$
41. $5b^4 + 2b^3 + 19b^2 - 2b + 21$ **43.** $x^3 + 6x^2y + 12xy^2 + 8y^3$
45. $64a^3 - 48a^2b + 12ab^2 - b^3$ **47.** $2a^3 - a^2b - 8ab^2 + 4b^3$
49. $2a^2 + 6a + 29$ **51.** $2x^2 + 21x - 26$ **53.** $y^2 + y + 8$
55. $-6b + 9$ **57.** $-14x^2 - 4x$ **59.** $26x - 28$
61. $a^{n+2} + 3a^n$ **63.** $3x^{2n} + x^n$ **65.** $x^{2n+3} - x^n$
67. $b^{2n} - 5b^n + 6$ **69.** $9x^{2n} + 6x^ny^n + y^{2n}$ **71.** $a^{2n-b^{6n}}$
73. $\pi R^2 - \pi r^2$ **75.** $2\pi rh + 2\pi r^2$ **77.** $\dfrac{Wx^4}{24EI} - \dfrac{\ell Wx^3}{16EI} - \dfrac{\ell^3 Wx}{48EI}$

Solutions to trial exercise problems

8. $-a^3b(3a^2b^5 - ab^4 - 7a^2b) = -3a^5b^6 + a^4b^5 + 7a^5b^2$
30. $(a - 3b)(a + 3b) = (a)^2 - (3b)^2 = a^2 - 9b^2$
38. $(x^2 - 2x + 1)(x^2 + 3x + 2) = x^4 + 3x^3 + 2x^2 - 2x^3 - 6x^2 - 4x + x^2 + 3x + 2 = x^4 + x^3 - 3x^2 - x + 2$ **42.** $(a - 3b)^3 = (a - 3b)(a - 3b)(a - 3b) = [(a - 3b)(a - 3b)](a - 3b) = [a^2 - 3ab - 3ab + 9b^2](a - 3b) = [a^2 - 6ab + 9b^2](a - 3b) = a^3 - 3a^2b - 6a^2b + 18ab^2 + 9ab^2 - 27b^3 = a^3 - 9a^2b + 27ab^2 - 27b^3$ **56.** $-2[5a - (2a + 3) - 3(3a + 7)] = -2[5a - 2a - 3 - 9a - 21] = -2[-6a - 24] = 12a + 48$
64. $x^{n+2}(x^{n+1} + x) = x^{n+2+n+1} + x^{n+2+1} = x^{2n+3} + x^{n+3}$
68. $(2a^n - b^n)^2 = (2a^n - b^n)(2a^n - b^n) = 4a^{n+n} - 2a^nb^n - 2a^nb^n + b^{n+n} = 4a^{2n} - 4a^nb^n + b^{2n}$

Review exercises

1. -10 **2.** 21 **3.** 4 **4.** -16 **5.** a^8 **6.** x^{12} **7.** x^6
8. $4a^2b^6$

Exercise 3–3

Answers to odd-numbered problems

1. 1 **3.** 1 **5.** 7 **7.** $\dfrac{1}{a^4}$ **9.** $\dfrac{3}{a^2}$ **11.** $\dfrac{a}{b^4c^3}$ **13.** $\dfrac{x^5}{2}$ **15.** $\dfrac{1}{a^8}$
17. $4x^3y^6$ **19.** $\dfrac{1}{27x^3}$ **21.** $\dfrac{1}{4}$ **23.** $-\dfrac{1}{81}$ **25.** $\dfrac{1}{4}$ **27.** $\dfrac{1}{x^3}$
29. $\dfrac{20}{x^5}$ **31.** $\dfrac{10b}{a}$ **33.** $\dfrac{1}{9x^8}$ **35.** $\dfrac{a^9}{64b^{12}}$ **37.** $\dfrac{8}{a^6}$ **39.** $\dfrac{b^9}{27a^6}$
41. $\dfrac{27a^6}{b^3}$ **43.** $\dfrac{27x^{12}y^{15}}{z^{27}}$ **45.** $\dfrac{a^{12}}{8b^{21}}$ **47.** $\dfrac{a^6}{b^7}$ **49.** $\dfrac{x^2}{y^3}$ **51.** $\dfrac{4}{x^4}$
53. $\dfrac{18}{a^7b^3}$ **55.** $\dfrac{1}{x^4z^8}$ **57.** $\dfrac{27x^9z^6}{y^{15}}$ **59.** $8a^3b^3$ **61.** $\dfrac{b^{13}c^8}{a^{12}}$
63. $\dfrac{y^6}{x^8z^8}$ **65.** a^{3b-5} **67.** x^{6n-8} **69.** a^{n+4} **71.** $a^{n+1}b^{2n+3}$
73. x^{-3n+3} **75.** 1.55×10^5 **77.** 8.63×10^{-2} **79.** 8.06×10^{21}
81. 5.787×10^{-4} **83.** 2.2046×10^{-3} **85.** 1.102×10^{-3}
87. 6.696×10^8 **89.** -0.0437 **91.** 4,990,000 **93.** 48,300
95. 3.17×10^{11} **97.** 1.25×10^{-9} **99.** 1.83×10^1
101. 1.19×10^{14} **103.** 2.52×10^{-6} **105.** 5.93×10^5
107. 3.18×10^9

Solutions to trial exercise problems

9. $3a^{-2} = 3 \cdot \dfrac{1}{a^2} = \dfrac{3}{a^2}$ **12.** $\dfrac{1}{4a^{-3}} = \dfrac{1}{4 \cdot \dfrac{1}{a^3}} = \dfrac{1}{\dfrac{4}{a^3}} = 1 \cdot \dfrac{a^3}{4} = \dfrac{a^3}{4}$

22. $-2^{-2} = -(2^{-2}) = -\left(\dfrac{1}{2^2}\right) = -\dfrac{1}{4}$

28. $(2a^{-3})(3a^{-5}) = 2 \cdot 3 \cdot a^{-3+(-5)} = 6a^{-8} = 6 \cdot \dfrac{1}{a^8} = \dfrac{6}{a^8}$

44. $\left(\dfrac{2x^5y^2}{4x^3y^7}\right)^4 = \left(\dfrac{x^2}{2y^5}\right)^4 = \dfrac{(x^2)^4}{(2y^5)^4} = \dfrac{x^8}{2^4(y^5)^4} = \dfrac{x^8}{16y^{20}}$

51. $\left(\dfrac{3x^{-2}}{12x^{-4}}\right)\left(\dfrac{16x^{-5}}{x}\right) = \left(\dfrac{1}{4} \cdot x^{(-2)-(-4)}\right)(16x^{(-5)-1}) = \left(\dfrac{1}{4}x^2\right)\left(16x^{-6}\right) = \dfrac{x^2}{4} \cdot \dfrac{16}{x^6} = \dfrac{4}{x^4}$

58. $(3x^2y^{-2})^2(x^{-4}y^3)^3 = \left(\dfrac{3x^2}{y^2}\right)^2\left(\dfrac{y^3}{x^4}\right)^3 = \dfrac{3^2(x^2)^2}{(y^2)^2} \cdot \dfrac{(y^3)^3}{(x^4)^3} = \dfrac{9x^4}{y^4} \cdot \dfrac{y^9}{x^{12}} = \dfrac{9y^5}{x^8}$ **89.** $-4.37 \times 10^{-2} = -0.0437$

Review exercises

1. $12a$ **2.** $3x^2$ **3.** $6ab$ **4.** $2a^3 + 3a^2$
5. $6a^4b + 9a^3b^2 - 3a^2b^3$ **6.** $ax + 2bx - 3ay - 6by$
7. $3ax + 4ay - 6bx - 8by$ **8.** $ab + a + b + 1$

Exercise 3–4

Answers to odd-numbered problems

1. $2^2 \cdot 3$ **3.** $2^3 \cdot 3$ **5.** $2^3 \cdot 7$ **7.** prime **9.** $3 \cdot 13$
11. $-(-2a + 3b)$ **13.** $-3(-a^3 + 3b^2)$
15. $-4(x^3 + 9xy - 4xy^2)$ **17.** $-3a^2b^2(-a - 4b - 5a^2)$
19. $-3x(-x + 3y)$ **21.** $-8R(3S + 2 - 4R)$
23. $5(x^2 + 2xy - 4y)$ **25.** $3a(6b - 9 + c)$ **27.** $3(5R^2 - 7S^2 + 12T)$ **29.** $3RS(R - 2S + 4)$ **31.** $4x^3y(3xy^2 - 2xy + 4)$
33. $3xy(x - 2y^3 + 5x^2y)$ **35.** $(3a + b)(x - y)$
37. $7(2b - 1)(a + c)$ **39.** $4a(b - 2c)(2x + y)$
41. $(5a - 1)(b + 3)$ **43.** $x^n(y^2 + z)$ **45.** $x^ny^n(x^ny^n - 1)$
47. $y^3(y^n - 1)$ **49.** $(a + 3b)(2x - y)$ **51.** $(2a + 3b)(x + 4y)$
53. $(2a - b)(x^2 + 3)$ **55.** $(c - 2d)(3a + b)$
57. $(2x^2 + 3)(x + 5)$ **59.** $(2x - 1)(4x^2 + 3)$
61. $\pi r(s + r)$ **63.** $P(1 + r)$

Solutions to trial exercise problems

4. $28 = 2 \cdot 2 \cdot 7 = 2^2 \cdot 7$ **18.** $-ab^2 - ac^2 = (-a)b^2 + (-a) \cdot c^2 = -a(b^2 + c^2)$ **36.** $5a(3x - 1) + 10(3x - 1) = \underline{5(3x - 1)} \cdot a + \underline{5(3x - 1)} \cdot 2 = 5(3x - 1)(a + 2)$ **46.** $x^{n+4} + x^4 = x^4 \cdot x^n + x^4 = x^4(x^n + 1)$ **53.** $2ax^2 - 3b - bx^2 + 6a = 2ax^2 - bx^2 + 6a - 3b = (2ax^2 - bx^2) + (6a - 3b) = x^2(2a - b) + 3(2a - b) = (2a - b)(x^2 + 3)$

Review exercises

1. $a^2 + 7a + 12$ **2.** $x^2 - 7x + 10$ **3.** $x^2 - 2x - 24$
4. $x^2 + 67x - 210$ **5.** $x^2 - 16$ **6.** $a^2 - 25$ **7.** $x^2 + 6x + 9$
8. $x^2 - 8x + 16$

Exercise 3-5

Answers to odd-numbered problems

1. $(x + 15)(x - 2)$ **3.** $(y + 8)(y - 3)$ **5.** $(x - 6)(x + 4)$
7. $3(x + 12)(x - 1)$ **9.** $4(x - 3)(x + 2)$
11. $3(y^2 - 9y + 4)$ **13.** $(xy + 3)(xy - 7)$
15. $(xy + 1)(xy + 12)$ **17.** $(xy - 2)(xy - 12)$
19. $4(ab + 4)(ab + 2)$ **21.** $-2(xy - 5)(xy + 2)$
23. $(a + 5b)(a + 2b)$ **25.** $(x + 3y)(x + 8y)$
27. $(a + 7b)(a - 5b)$ **29.** $(x + 8y)(x + 2y)$
31. $(x^n + 7)(x^n + 2)$ **33.** $(x^n - 5)(x^n - 3)$ **35.** $(b - 6)^2$
37. $(c + 9)^2$ **39.** $(3y + 2)^2$ **41.** $(x - 7y)^2$ **43.** $(2x - 3y)^2$
45. $(3x + 5y)^2$ **47.** $(a - 2b)(x + 6)(x + 2)$
49. $(y + 2z)(x - 10)(x - 3)$ **51.** $(3y - z)(x + 12)(x + 3)$
53. $(3x - y)(a - 12)(a + 5)$ **55.** $(a + b - 6)(a + b + 2)$
57. $(y + 3z - 7)(y + 3z + 2)$ **59.** $(R + 5S - 4)(R + 5S - 2)$
61. $(W - 17)(W - 9)$ **63.** $(C_2 + 7)(C_2 - 6)$

Solutions to trial exercise problems

10. $5y^2 - 15y - 55 = 5(y^2 - 3y - 11)$; m and n do not exist, therefore $5(y^2 - 3y - 11)$ is the completely factored form.
21. $-2x^2y^2 + 6xy + 20 = -2(x^2y^2 - 3xy - 10)$; m and n are -5 and 2. Then $-2(x^2y^2 - 3xy - 10) = -2(xy - 5)(xy + 2)$.
23. $a^2 + 7ab + 10b^2$; m and n are 5 and 2. $(a + 5b)(a + 2b)$
30. $x^{2n} + 5x^n + 6$; m and n are 2 and 3; $(x^n + 2)(x^n + 3)$
47. $x^2(a - 2b) + 8x(a - 2b) + 12(a - 2b) = (a - 2b)(x^2 + 8x + 12)$. In the second parentheses m and n are 6 and 2, and we have $= (a - 2b)(x + 6)(x + 2)$. **55.** $(a + b)^2 - 4(a + b) - 12$; m and n are -6 and 2. $[(a + b) - 6][(a + b) + 2]$ $= (a + b - 6)(a + b + 2)$

Review exercises

1. $(2x - 1)(3x + 5)$ **2.** $(3a + 1)(2a + 1)$ **3.** $(2x - 3)(4x - 1)$ **4.** $(x + 4)(5x - 3)$ **5.** $6x^2 + 7x + 2$
6. $10a^2 + 13a - 3$ **7.** $3b^2 + 5b - 28$ **8.** $9a^2 - 4$

Exercise 3-6

Answers to odd-numbered problems

1. $(3a - 2)(a + 3)$ **3.** $(4x - 1)(x - 1)$ **5.** $(x + 2)(x + 16)$
7. $(2y + 1)(y - 1)$ **9.** $(8x - 1)(x - 2)$ **11.** $(2a - 3)(a - 4)$
13. $(2x + 9)(x + 2)$ **15.** $(7x - 1)(x + 3)$
17. $(2x + 1)(2x - 3)$ **19.** $(6x + 1)(x - 4)$
21. $(2z + 1)(5z + 2)$ **23.** will not factor
25. $(5x + 1)(x - 2)$ **27.** $(3x - 4)(2x - 3)$
29. $3(x + 2)(x + 2) = 3(x + 2)^2$ **31.** will not factor
33. $2(3x - 4)(x - 5)$ **35.** $(2x + 3)(-2x - 3) = -(2x + 3)^2$
37. $(-7x + 1)(x - 5)$ or $(7x - 1)(-x + 5)$ or $-(7x - 1)(x - 5)$ **39.** $(3x + 4)(4x - 1)$
41. $2x(x + 2)(2x + 1)$ **43.** $2(3x - 8)(3x + 1)$
45. $(4x - 3)(2x - 3)$

Solutions to trial exercise problems

26. $6x^2 + 21x + 18 = 3(2x^2 + 7x + 6)$; m and n are 3 and 4. $3[2x^2 + 3x + 4x + 6] = 3[(2x^2 + 3x) + (4x + 6)]$ $= 3[x(2x + 3) + 2(2x + 3)] = 3(2x + 3)(x + 2)$
35. $-4x^2 - 12x - 9$; m and n are -6 and -6. $-4x^2 - 6x - 6x - 9 = (-4x^2 - 6x) + (-6x - 9) = -2x(2x + 3) - 3(2x + 3) = (2x + 3)(-2x - 3) = -(2x + 3)^2$
41. $4x^3 + 10x^2 + 4x = 2x(2x^2 + 5x + 2)$ m and n are 4 and 1. $2x(2x^2 + 4x + x + 2) = 2x[(2x^2 + 4x) + (x + 2)]$ $= 2x[2x(x + 2) + 1(x + 2)] = 2x(x + 2)(2x + 1)$

Review exercises

1. $9a^2 - b^2$ **2.** $x^2 - 4y^2$ **3.** $9x^2 - 16y^2$ **4.** $x^4 - 25$
5. $x^4 - 16$ **6.** $a^3 + 8b^3$ **7.** $x^3 - 27y^3$ **8.** $27a^3 - 8b^3$

Exercise 3-7

Answers to odd-numbered problems

1. $(a + 7)(a - 7)$ **3.** $(8 + S)(8 - S)$ **5.** $(6x + y^2)(6x - y^2)$
7. $(4a + 7b)(4a - 7b)$ **9.** $8(a + 2b)(a - 2b)$
11. $2(5 + x)(5 - x)$ **13.** $2(7ab + 5xy)(7ab - 5xy)$
15. $(a^2 + 9)(a + 3)(a - 3)$ **17.** $(x^2 + 4y^2)(x + 2y)(x - 2y)$
19. $(a^n + 2)(a^n - 2)$ **21.** $(x^n + y^n)(x^n - y^n)$
23. $(x^{2n} + 9)(x^n + 3)(x^n - 3)$ **25.** $(a + 5b)(2x + y)(2x - y)$
27. $3(3x - y)(a + 3)(a - 3)$ **29.** $3(3x - y)(2a + b)(2a - b)$
31. $(2a + b + x - 2y)(2a + b - x + 2y)$
33. $(3a - b + 2a - b)(3a - b - 2a + b) = (5a - 2b)(a)$ $= a(5a - 2b)$ **35.** $(x + 2y + x - 3y)(x + 2y - x + 3y)$ $= (2x - y)(5y) = 5y(2x - y)$ **37.** $(3a - b)(9a^2 + 3ab + b^2)$
39. $(x + y)(x^2 - xy + y^2)$ **41.** $(a + 2b)(a^2 - 2ab + 4b^2)$
43. $(2y - 3x)(4y^2 + 6xy + 9x^2)$
45. $3(3a - b)(9a^2 + 3ab + b^2)$ **47.** $3(x + 2)(x^2 - 2x + 4)$
49. $(4z + 5)(16z^2 - 20z + 25)$
51. $2(a - 3b)(a^2 + 3ab + 9b^2)$ **53.** $R^2(R + 4S)(R^2 - 4RS + 16S^2)$ **55.** $3x^2(x - 3y)(x^2 + 3xy + 9y^2)$
57. $(xy^4 + z^3)(x^2y^8 - xy^4z^3 + z^6)$ **59.** $(a^6b^3 - 3c)(a^{12}b^6 + 3a^6b^3c + 9c^2)$ **61.** $3(xy^2 + 3z)(x^2y^4 - 3xy^2z + 9z^2)$
63. $(x + 3y + z)(x^2 + 6xy + 9y^2 - xz - 3yz + z^2)$
65. $(4a + b - 3c)(16a^2 + 8ab + b^2 + 12ac + 3bc + 9c^2)$
67. $(a - 2b - 2x - y)(a^2 - 4ab + 4b^2 + 2ax - 4bx + ay - 2by + 4x^2 + 4xy + y^2)$ **69.** $(5x + 2y)(7x^2 - xy + 13y^2)$
71. $9x(x^2 - xy + y^2)$ **73. a.** $\dfrac{V}{8I}(h + 2v_1)(h - 2v_1)$, **b.** $\dfrac{3V}{2A}\left(1 + \dfrac{2V_1}{H}\right)\left(1 - \dfrac{2V_1}{H}\right)$

Solutions to trial exercise problems

9. $8a^2 - 32b^2 = 8(a^2 - 4b^2) = 8[(a)^2 - (2b)^2] = 8(a + 2b)(a - 2b)$ **14.** $x^4 - y^4 = (x^2)^2 - (y^2)^2 = (x^2 + y^2)(x^2 - y^2)$ $= (x^2 + y^2)(x + y)(x - y)$ **18.** $x^{2n} - 1 = (x^n)^2 - (1)^2$ $= (x^n + 1)(x^n - 1)$ **24.** $a^2(x + 2y) - b^2(x + 2y)$ $= (x + 2y)(a^2 - b^2) = (x + 2y)(a + b)(a - b)$ **31.** $(2a + b)^2 - (x - 2y)^2 = [(2a + b) + (x - 2y)][(2a + b) - (x - 2y)]$ $= (2a + b + x - 2y)(2a + b - x + 2y)$
44. $64a^3 - 8 = 8(8a^3 - 1) = 8[(2a)^3 - (1)^3]$.
Then $8(\quad - \quad)[(\quad)^2 + (\quad)(\quad) + (\quad)^2]$ and $8(2a - 1)$ $[(2a)^2 + (2a)(1) + (1)^2] = 8(2a - 1)(4a^2 + 2a + 1)$.
55. $3x^5 - 81x^2y^3 = 3x^2(x^3 - 27y^3) = 3x^2[(x)^3 - (3y)^3]$.
Then $3x^2(\quad - \quad)[(\quad)^2 + (\quad)(\quad) + (\quad)^2]$ and $3x^2(x - 3y)$ $[(x)^2 + (x)(3y) + (3y)^2] = 3x^2(x - 3y)(x^2 + 3xy + 9y^2)$.
62. $(a + 2b)^3 - c^3$. Then $(\quad - \quad)[(\quad)^2 + (\quad)(\quad) + (\quad)^2]$ and $[(a + 2b) - c]\ [(a + 2b)^2 + (a + 2b)(c) + (c)^2]$ $= (a + 2b - c)(a^2 + 4ab + 4b^2 + ac + 2bc + c^2)$.

Review exercises

1. $(a - 5)(a - 2)$ **2.** $(5a + 2b)(x - 2y)$
3. $(x + 2y)^2$ **4.** $(3a + b)(2a - b)$ **5.** $5a(a - 3)(a - 5)$
6. $(3x - 4)(2x + 3)$

Exercise 3–8

Answers to odd-numbered problems

1. $(m-7)(m+7)$ **3.** $(x+5)(x+1)$ **5.** $(7a+1)(a+5)$
7. $(2a+3)(a+6)$ **9.** $(ab+4)(ab-2)$
11. $(3a+b)(9a^2-3ab+b^2)$ **13.** $5(3x+y)(5x^2+a)$
15. $10(x-y)^2$ **17.** $4(m-2n)(m+2n)$ **19.** $(a-b-2x-y)$ $(a-b+2x+y)$ **21.** $3(x^2-3y)(x^4+3x^2y+9y^2)$
23. $2xy^2(6x^2-9x+8y^2)$ **25.** $(2x+3y)(2x-3y)$
27. $(x+2y)(3a-b)$ **29.** $(3a^3-bc)(9a^6+3a^3bc+b^2c^2)$
31. $(5a+3)(a-7)$ **33.** $(a-1)(a+1)(a-2)(a+2)$
35. $(2a+3b)(2a-5b)$ **37.** $(y-2)(y+2)(y^2+4)$
39. $2(a+2)(2a+1)$ **41.** $(x+y-9)(x+y+1)$
43. $(2a-1)(3a+5)$ **45.** $4ab(x+3y)(1-2ab)$
47. $(2a-5b)^2$ **49.** $5x(4x^2+1)(2x+1)(2x-1)$
51. $3ab(a^2-3b^2)^2$ **53.** $(3a-x-5y)(3a+x+5y)$
55. $(3x-13)(x+7)$ **57.** $3x^2(xy^3+3z^2)(x^2y^6-3xy^3z^2+9z^4)$
59. $(3-x)(3+x)(a-3)^2$

Review exercises

1. $\{7\}$ **2.** $\left\{-\frac{3}{2}\right\}$ **3.** $\left\{\frac{4}{5}\right\}$ **4.** $\{0\}$ **5.** $\left\{\frac{6}{5}\right\}$ **6.** $\{2\}$

Chapter 3 review

1. $-15x^5$ **2.** $6a^3b^5$ **3.** $8a^9b^{12}c^3$ **4.** $x^8y^9z^2$ **5.** $17a^7$
6. $4b^{10}-27b^8$ **7.** $10a^4b-15a^3b^2+20a^2b^3$ **8.** $2a^2+ab-b^2$
9. y^2-49 **10.** $4a^2+4ab+b^2$ **11.** $a^3-2a^2b-ab^2+2b^3$
12. $2x^2+16x-17$ **13.** $\frac{b^3}{4a^3}$ **14.** $\frac{6}{x^9}$ **15.** $\frac{9y^6}{x^4}$ **16.** $\frac{64x^9}{y^{15}}$
17. $\frac{a^9}{b^3}$ **18.** $\frac{a^3c^7}{2b}$ **19.** $\frac{4}{y^5}$ **20.** $6x^2(2x-3)$
21. $4a^2b(3a^2-b^2+6ab)$ **22.** $(x-2y)(a+b)$
23. $5x(2x+1)(x-3z)$ **24.** $(2x-y)(3a-b)$
25. $(2y+x)(4a+3b)$ **26.** $(2x+y)(2a+3b)$
27. $(a+3b)(x-2y)$ **28.** $(2x-y)(3a-b)$
29. $(a+3b)(2x+y)$ **30.** $(a+2)(a+12)$
31. $(b-7)(b-2)$ **32.** $2a(a-5)(a+1)$
33. $x(x-3)(x+2)$ **34.** $(xy+6)(xy+4)$
35. $(ab-10)(ab+2)$ **36.** $(x+3y-1)(x+3y+2)$
37. $(x+2y-7)^2$ **38.** $(a-2b)(a+b)$
39. $(x+2y)(x+3y)$ **40.** $(2a-3)(a-2)$
41. $(4x-5)(2x-1)$ **42.** $(2y+1)(3y-4)$
43. $(3a-1)(a+1)$ **44.** $(4x-1)(x+3)$
45. $(2x+1)(2x+1)=(2x+1)^2$ **46.** $(6a+1)(4a+3)$
47. $(4x-3)(2x-3)$ **48.** $(2x+3)(x+6)$
49. $3(2x+1)(x+8)$ **50.** $(-2x-1)(x-6)$ or $(2x+1)(-x+6)$ or $-(2x+1)(x-6)$ **51.** $(x+9)(x-9)$
52. $4(a+3b)(a-3b)$ **53.** $3(a+3b)(a-3b)$
54. $(y^2+9)(y+3)(y-3)$ **55.** $(a+2b)(x+y)(x-y)$
56. $(x-3z)(2a+3b)(2a-3b)$ **57.** $(a+2b)(a^2-2ab+4b^2)$
58. $(3x-y)(9x^2+3xy+y^2)$ **59.** $8(2a+b)(4a^2-2ab+b^2)$
60. $x^2(3x+y)(9x^2-3xy+y^2)$ **61.** $2(x-2b)(x^2+2bx+4b^2)$
62. $(x^4y^5-z^3)(x^8y^{10}+x^4y^5z^3+z^6)$ **63.** $(3x+4)^2$
64. $(a^2-b^3)(a^4+a^2b^3+b^6)$ **65.** $3a(a+5)(a-5)$
66. $(a+6)^2(3+x)(3-x)$ **67.** $(6a-1)(a+3)$
68. $(x-2y)(4m+3n)$ **69.** $5x(2x+y)(4x^2-2xy+y^2)$
70. $ab^3(a-2b)^2$ **71.** $(x+4y)(x-2y)$
72. $(3a+2)(5a-2)$

Chapter 3 cumulative test

1. false **2.** false **3.** true **4.** 2 **5.** undefined **6.** 4 **7.** 0
8. 81 **9.** 37 **10.** -34 **11.** $\{x|x \geq 7\}$ **12.** $\{5\}$
13. $\left\{-3,\frac{1}{3}\right\}$ **14.** $\left\{x|-\frac{1}{3}<x<\frac{17}{3}\right\}$
15. $\left\{x|x<\frac{-11}{6} \text{ or } x>\frac{1}{6}\right\}$ **16.** $\left\{x|-\frac{3}{2}<x<\frac{7}{2}\right\}$
17. $\left\{x|-5 \leq x \leq \frac{-3}{2}\right\}$ **18.** let x = the number; $\{x|x>9\}$
19. \$4,000 **20.** $6a^5b^5$ **21.** $16x^{12}y^{16}$
22. $\frac{3a^3b^5}{2c^6}$ **23.** $\frac{y^4}{x^6}$ **24.** $\frac{y^9}{27x^9}$ **25.** $4x^2-2x-3$ **26.** $\frac{a^9c^3}{27b^6}$
27. $7ab-a-5b$ **28.** $9a^2-6ab+b^2$ **29.** $8a^{18}b^{11}$
30. $12x^6y^3-18x^6y^4+24x^4y^5$ **31.** $(x+6)(x-6)$
32. $(x+3y)(x^2-3xy+9y^2)$ **33.** $(2a-3)(a-6)$
34. $(y+6)(y+1)$ **35.** $(xy+2)(xy-4)$
36. $(7x+1)(x-5)$ **37.** $(2a+3b)(x-3y)$
38. $5x(2a-b)(3x+1)$ **39.** $(2x-5y)^2$
40. $3(a+2b)(a-2b)$

Chapter 4

Exercise 4–1

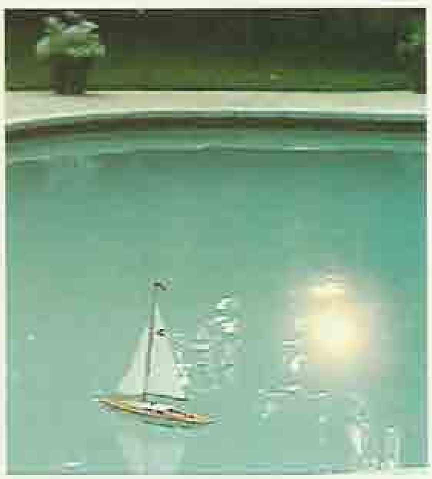
Answers to odd-numbered problems

1. domain $=\{x|x \in R, x \neq 4\}$ **3.** domain $=\{x|x \in R, x \neq -1\}$
5. domain $=\left\{z|z \in R, z \neq \frac{3}{2}\right\}$ **7.** domain $=\left\{x|x \in R, x \neq -\frac{4}{7}\right\}$
9. domain $=\{x|x \in R, x \neq 0,4\}$ **11.** domain $=\{x|x \in R, x \neq -4\}$
13. domain $=\left\{y|y \in R, y \neq -\frac{5}{2},\frac{5}{2}\right\}$
15. domain $=\left\{x|x \in R, x \neq -\frac{1}{2},3\right\}$
17. domain $=\{x|x \in R\}$ **19.** $\frac{5}{3x}\ (x \neq 0)$ **21.** $\frac{p^2}{q^2}\ (p \neq 0, q \neq 0)$
23. $-\frac{ab^3}{c^4}\ (a \neq 0, b \neq 0, c \neq 0)$ **25.** $\frac{3n^2p^2}{4m^3}\ (m \neq 0, n \neq 0, p \neq 0)$
27. $\frac{3}{7}\ (x \neq 2)$ **29.** $\frac{3}{a-2}\ (a \neq 0,2)$ **31.** $2(x-1)\ (x \neq 0)$
33. $\frac{4(y+1)}{3}\ (y \neq 1)$ **35.** $-5\ (x \neq y)$ **37.** $\frac{a-3}{4}\ (a \neq -3)$
39. $2y-1\ \left(y \neq -\frac{1}{2}\right)$ **41.** $x^2-2x+4\ (x \neq -2)$
43. $\frac{-3}{2(y^2+xy+x^2)}\ (x \neq y)$ **45.** $\frac{y-7}{y+7}\ (y \neq -7)$
47. $\frac{m-6}{m-3}\ (m \neq -2,3)$ **49.** $\frac{y-8}{y-5}\ (y \neq -4,5)$
51. $\frac{a-1}{a+1}\ \left(a \neq -1,\frac{1}{2}\right)$ **53.** $\frac{2x-3}{4x+1}\ \left(x \neq -3,-\frac{1}{4}\right)$
55. $\frac{3m+2}{2m+1}\ \left(x \neq -\frac{3}{4},-\frac{1}{2}\right)$ **57.** $\frac{2(a+2)}{3a+1}\ \left(a \neq -\frac{1}{3},\frac{3}{2}\right)$
59. $\frac{5(y+1)}{4(y+3)}\ (y \neq -3,3)$ **61.** $\frac{-(a+6)}{3+a}\ \left(a \neq -3,\frac{2}{3}\right)$
63. $\frac{p+4q}{p+2q}\ (p \neq -3q,-2q)$

Contents

Chapter 3 ■ Exponents and Polynomials

Chapter 4 ■ Rational Expressions



Chapter 5 ■ Exponents, Roots, and Radicals

Chapter 6 ■ Quadratic Equations and Inequalities

Chapter 7 ■ Linear Equations and Inequalities in Two Variables

Chapter 8 ■ Systems of Linear Equations

Chapter 9 ■ Conic Sections

Chapter 10 ■ Functions

Chapter 11 ■ Exponential and Logarithmic Functions

Chapter 12 ■ Sequences and Series

Index